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# Metrics, Network Traffic, Performance

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## Outline

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- Define Network Performance Metrics
- Application Requirements
- Traffic Models
- Theoretical Prediction of Delay and Loss
- Simulation of Networks

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# Delay

## □ Delay

### ➤ Propagation delay = $D/s$

- $D(m)$  Distance between NEs
- $s$  propagation speed in media
  - $s = 3 \times 10^8$  (m/s) in free space
  - $s = (2/3)c = 2 \times 10^8$  (m/s) in fiber
  - $s = 0.88c = 2.64 \times 10^8$  (m/s) in coax

### ➤ Transmission delay (clocking time) = $L/R$

- $L$  = Packet Length (bits)
- $L$  is random
- $R$  = Link rate (bits/sec)

### ➤ Arrival times of packets are random

### ➤ Queueing delay = random

### ➤ Processing delay, e.g.,

- Error check
- Read destination address
- Forwarding
- Special handling
- Common to assume  
Processing delay  $\ll$  other delays

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# Delay

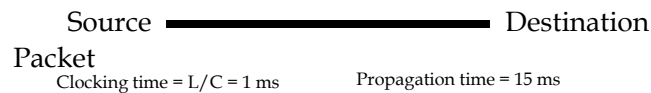
## □ Propagation Delay:

### ➤ *The Speed of Light Limitation*

$$\text{Propagation delay} = \text{Distance (m)} / (\text{Speed of Light m/s})$$

- Example: 3000 km fiber link  
Speed of light in fiber  $= s = 0.66 \times (3 \times 10^8 \text{ m/s})$   
Propagation delay =  $3000 \times 10^3 \text{ m} / 0.66 \times (3 \times 10^8 \text{ m/s}) = 15 \text{ ms}$
- +Other Media
  - Speed of light in free space =  $1.0 \times (3 \times 10^8 \text{ m/s})$
  - Speed of light in coax =  $0.88 \times (3 \times 10^8 \text{ m/s})$
- Effect of clocking time,  $L/R$  "putting the bits on the link"

Example: Distance = 3000km, Data rate = 1 Mb/s, Packet size = 1000 bits

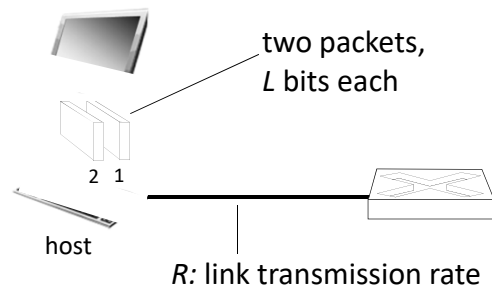


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# Transmission Delay

- transmits packet into access network at *transmission rate R*
  - link transmission rate  $R$  (b/s), aka link *capacity*
  - Bandwidth (Hz)  $\neq R$  (b/s)*

Example:  $L=1000$  Bytes,  $R=10\text{Mb/s}$ , clocking delay= 0.8ms

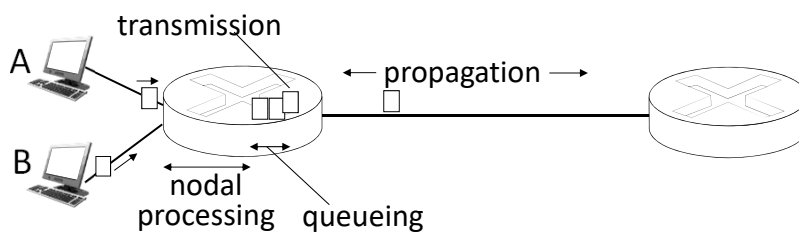


$$\text{packet transmission delay} = \text{time needed to transmit } L\text{-bit packet into link} = \frac{L \text{ (bits)}}{R \text{ (bits/sec)}}$$

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# Packet delay: four sources



$$d_{\text{nodal}} = d_{\text{proc}} + d_{\text{queue}} + d_{\text{trans}} + d_{\text{prop}}$$

$d_{\text{proc}}$ : nodal processing

- check bit errors
- determine output link
- typically < msec
- Often assumed to be zero

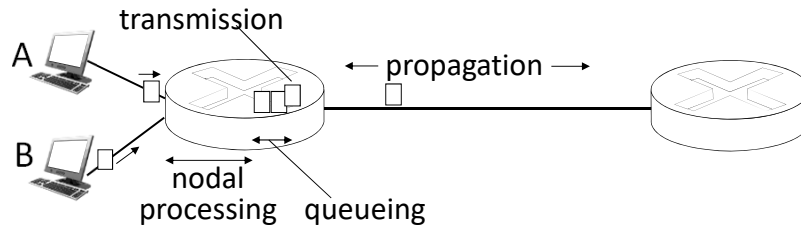
$d_{\text{queue}}$ : queueing delay

- time waiting at output link for transmission
- depends on congestion level of router

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## Packet delay: four sources



$$d_{\text{nodal}} = d_{\text{proc}} + d_{\text{queue}} + d_{\text{trans}} + d_{\text{prop}}$$

$d_{\text{trans}}$ : transmission delay:

- $L$ : packet length (bits)
- $R$ : link transmission rate (bps)

$$d_{\text{trans}} = L/R$$

$d_{\text{prop}}$ : propagation delay:

- $D$ : length of physical link
- $s$ : propagation speed

$$d_{\text{prop}} = D/s$$

$d_{\text{trans}}$  and  $d_{\text{prop}}$   
very different

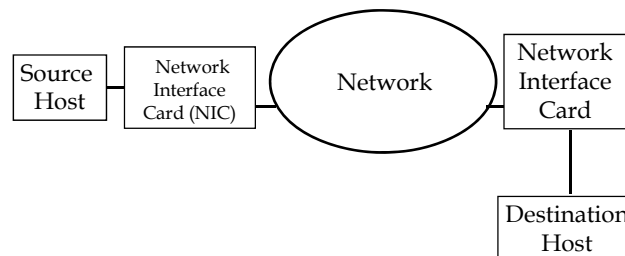
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## Response Time-Latency

Response time  $T_R$ : The time to “correctly” transmit a packet from Source to destination.

“correctly” implies Response time includes acknowledgments



Examine key components of delay

Switching, impairments, metrics ...

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## Delay vs Latency

- Latency and network delay both refer to the time it takes for data to travel from one point in a network to another.
- Latency is the time it takes for a packet of data to travel from its source to its destination and get a response, e.g, response time. Most common metric is the minimum RTT.
- Delay is sum of processing, queueing, transmission and propagation times.
- For more details see: "Internet Measurement: Infrastructure, Traffic and Applications" Mark Crovella, Balachander Krishnamurthy, Wiley, 2006

## Delay-Network Classifications

- PAN: Personal Area Networks [BAN: Body Area Network]
  - ~ 3 m or 10ns
- DAN.: Desk Area Networks
  - ~ 3 m or ~10ns
- **LAN: Local Area Networks**
  - ~3 km or ~10us
- **MAN: Metropolitan Area Networks**
  - ~300 km or ~1ms
- NAN: National Area Networks
  - ~3000 Km or ~10ms
- GAN: Global Area Networks
  - ~10,000 Km or 30ms
  - NANs and GANs are typically called WANs Wide Area Networks
- **Terrestrial Networks**
- **Satellite Networks**
  - Geosynchronous Earth Orbit (GEO)  
35,800 km ~120ms to satellite
    - One hop ~240ms
    - RTT ~480ms
  - Low Earth Orbit (LEO) 550km ~1.8 ms
    - One hop ~3.6ms
    - RTT ~7.2ms
- **Interplanetary Networks**

## Delay-Bandwidth Product

- Delay-Bandwidth Product
  - One way propagation delay =  $\tau$  sec
  - Round-trip-time (RTT) =  $2\tau$  sec
  - Link rate =  $R$  b/s
  - Delay-Bandwidth Product (bits) =  $2\tau R$  bits (# bits in RTT)
- Number of packets in Delay-Bandwidth Product
  - $L$  Packet length in bits/packet
  - Number of packets in Delay-Bandwidth Product =  
Number of packets in round trip time (RTT)  
= (Delay-Bandwidth Product-bits)/( $L$ -bits/packet)
- Example:
  - $D = 2000$  km
  - $c = 2 \times 10^8$  m/s (fiber)
  - $\tau = 10$  ms
  - $L = 1000$  Bytes
  - $R = 10$  Gb/s
  - Delay-Bandwidth Product = 200 Mb
  - # packets in DBP=25,000
  - The transmission line is “storing” 25,000 packets

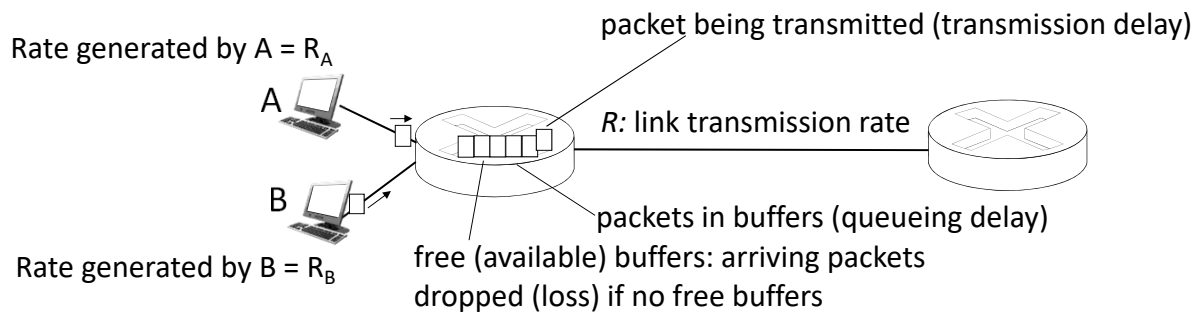
## Packet loss

- No space in Queue to store incoming packet: Cause-network congestions
- Corrupted packets: Cause-bit errors
- Networks may try to recover lost packets
- Or applications cope with lost packets, e.g., packet voice and video often accept a lost packet as an impairment.
- Some protocols “recover” lost packets
  - Data Link Control Protocols-DLCs (details later)
  - Transmission Control Protocol-TCP (details later)

# Packet loss

packets *queue* in router/switch buffers

- packets queue, wait for turn
- arrival rate to link  $R_A + R_B > R$  (temporarily) exceeds output link capacity leads to packet loss



Show [http://www.itc.ku.edu/~frost/E ECS\\_563/LOCAL/Extend\\_Models\\_2019-v10/Stat-Mux-Finite-System-Throughput-Blocking\\_Delay-ES10.mox](http://www.itc.ku.edu/~frost/E ECS_563/LOCAL/Extend_Models_2019-v10/Stat-Mux-Finite-System-Throughput-Blocking_Delay-ES10.mox)

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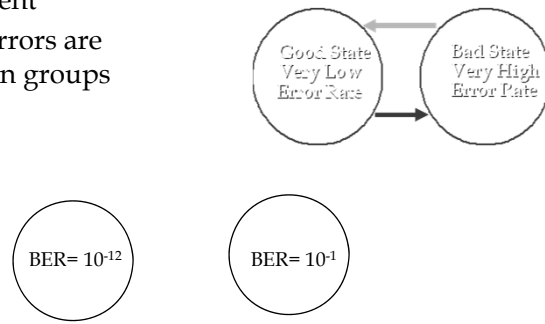
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# Bit Errors

- Bit errors can lead to packet loss, protocols do *error detection*
- Bit errors rate (BER) can lead to packet loss
- Lost packets can recovered by
  - Error correction
    - Forward Error Correction Coding (FEC)
  - Retransmission
  - Both FEC and Retransmission
- Error environment: The BER can range
  - Coaxial links:  $10^{-9}$  to  $10^{-6}$
  - Fiber optic links:  $10^{-12}$  (after FEC)
  - Wireless links:  $10^{-5}$  to  $10^{-3}$ ,
- BER depends on
  - Signal strength,
  - Signal quality (receiver cost),
  - Noise,
  - Interference,
  - Channel effects, e.g., fading

## Bit Errors

- Model 1: Random, bit errors are statistically independent
- Model 2: Bursty, bit errors are correlated and come in groups



## Bit Errors-time between errors

- Example:
  - Line rate = 600 Mb/s
  - Bit error rate( BER ) =  $10^{-9}$
- What is the time between errors?
  - On average see one error in  $10^9$  bits
  - $(10^9 \text{ bits/error}) / (600 \text{ Mb/s}) = 1.66 \text{ sec between errors}$



## Bit Errors

### □ Example: Impact of delay and errors:

➤ Link rate 600 Mb/s

➤ Free Space

➤ Link distance 3000 km  $\Rightarrow$  10ms

➤ Packet size:

- Payload 1000 bytes

750

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$$8000 \text{ bits} / (600 \text{ Mb/s}) = 13.3 \mu\text{s} / \text{packet}$$

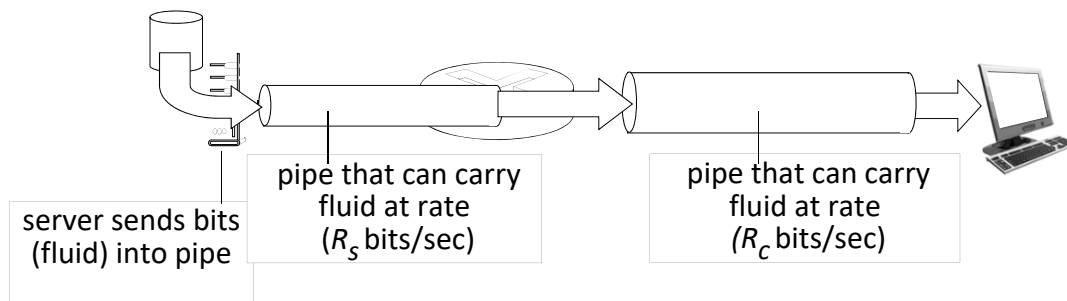
$$10 \text{ ms} / (13.3 \mu\text{s} / \text{packet}) = 750 \text{ packets in flight}$$

→ Many packets can be in transit.

Question: How do you cope with packets in error?

## Throughput

- *throughput*: rate (bits/time unit) at which bits are being sent from sender to receiver
  - *instantaneous*: rate at given point in time
  - *average*: rate over longer period of time



# Throughput

- **Throughput** in b/s or packets/sec,
- **Normalized throughput**

$R_{Ave}$  = Average error free rate (b/s) passing a reference point in the network

$R$  = Link Capacity (b/s) = Peak link rate

$S$  = % time the network is carrying error free packets-goodput

# Throughput measurements

**Table 3: Average/Variation Range of Application Throughput (Mbps) across different mobility patterns and network technologies (file download scenario only)**

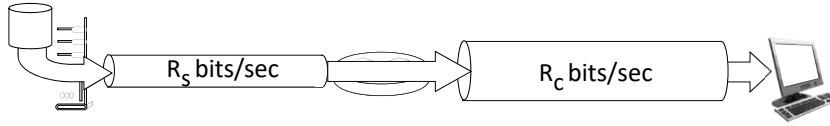
Network Technology	Mobility Patterns								
	Static				Car				
	Avg.	Var.	Range	# Traces	Trace Dur. (m)	Avg.	Var.	Range	# Traces
5G	66.9	(22.0, 202.5)	5	260	28.5	(3.0, 88.5)	16	459	
4G	42.6	(21.3, 77.2)	5	39	22.3	(3.2, 49.1)	12	290	

**Table 4: Average/Variation Range of Application Throughput (Mbps) across different mobility patterns and application types**

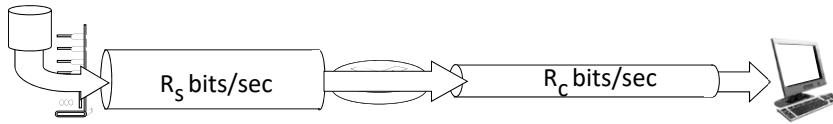
Application	Mobility Patterns								
	Static				Car				
	Avg.	Var.	Range	# Traces	Trace Dur. (m)	Avg.	Var.	Range	# Traces
File download	66.9	(22.0, 202.5)	5	260	28.5	(3.0, 88.5)	16	459	
Netflix	13.7	(0.5, 31.1)	10	576	7.5	(0.4, 19.9)	23	637	
Amazon Prime	6.9	(0.3, 11.2)	8	582	1.3	(0.3, 2.7)	21	628	

# Throughput

$R_s < R_c$  What is average end-end throughput?



$R_s > R_c$  What is average end-end throughput?

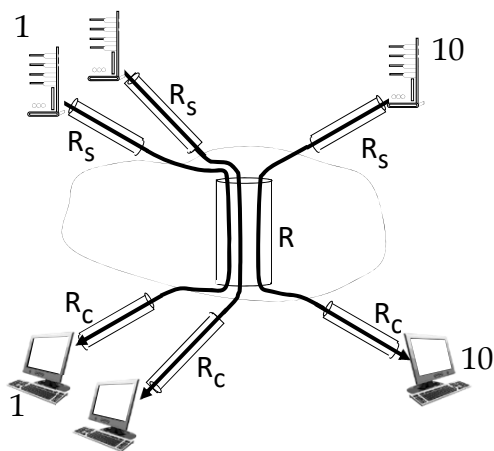


*bottleneck link*

link on end-end path that constrains end-end throughput

Modified from: 8<sup>th</sup> edition Jim Kurose, Keith Ross Pearson, 2020

# Throughput: network scenario



10 connections (fairly) share  
backbone bottleneck link  $R$  bits/sec

- per-connection end-end throughput:  $\min(R_c, R_s, R/10)$
- in practice:  $R_c$  or  $R_s$  is often bottleneck

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## Utilization

- **Channel (or link) utilization:**
  - The % time the channel (or link is busy)
- **Channel Efficiency**
  - The % time the channel is carrying user information (impact of overhead)

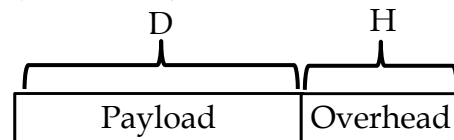
Example: Let

$D = \# \text{user data bits / packet} = \text{Payload}$

$H = \# \text{network overhead bits / packet} = \text{number of bits in the header}$

then

$$\text{Channel efficiency} = S \left( \frac{D}{D+H} \right)$$



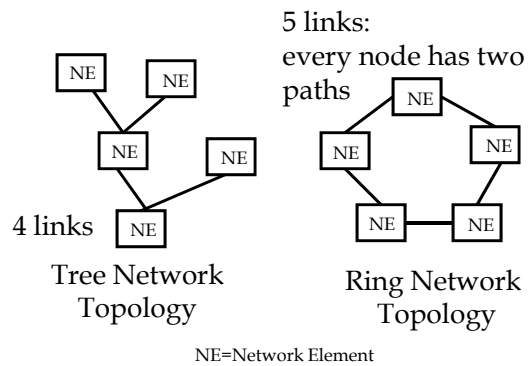
## Reliability

- **Reliability:** The reliability of a network can be defined as the probability that the functioning nodes are connected to working links.

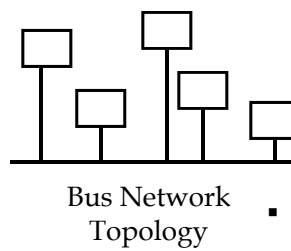
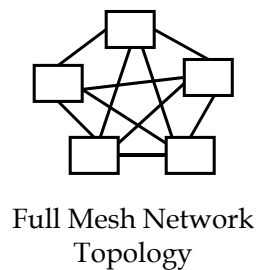
$$\text{Reliability} = 1 - \text{Network Failure}$$

- Here lets assume all nodes are working and analyze the “Reliability” of basic ring and tree networks where only links fail

## Reliability

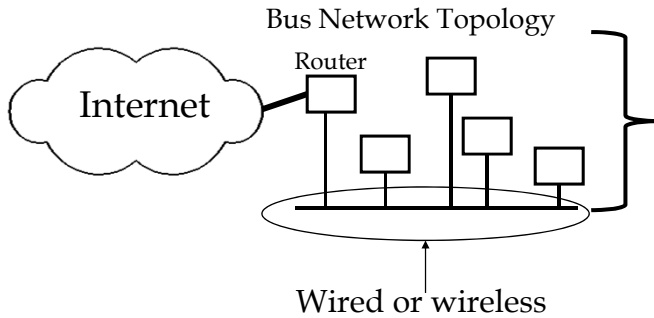


## Aside: Other Network Topologies



- Note a wireless network can be viewed as a bus topology
- All users hear all transmissions.
- Use of the transmission media coordinated using a MAC protocol.

## Aside: Other Network Topologies



Key concepts:

1) **Broadcast**

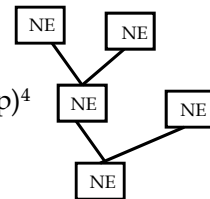
Everyone on the same "Network" can directly communicate, using a point-to-point link, DLC protocol.

2) Packet delivered to one special node, i.e., a router, connected to the "Network" is sufficient for delivery to any destination connected to the broadcast network.

## Network Performance Criteria: Reliability

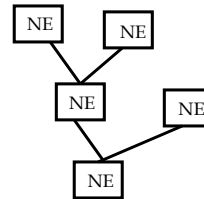
□ Example:

- Reliability for a 5 **node tree network**
- Any of the 4 links fail the network is down
- Let  $p$  = probability of link failure
- Assume failures are statistically independent
- $q = 1-p$  = probability of link operational
- Then Reliability = Prob[all links operational] =  $(1-p)^4$
- Prob[network failure] =  $1 - (1-p)^4$



## Network Performance Criteria: Reliability

- But  $(1-p)^4 = 1 - 4p + 6p^2 - 4p^3 + p^4$
- Prob[network failure] =  $4p - 6p^2 + 4p^3 - p^4$
- Assuming  $p$  is small then for 5 node tree network (4 links)
- Prob[network failure]  $\approx 4p$
- Reliability  $\approx 1-4p$



Switching, impairments, metrics ...

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## Network Performance Criteria: Reliability

- Reliability for a 5 node ring network
- Ring network has 5 links
- Ring network can have one link failure and still be working, note one more link can fail
- Let  $q = 1 - p =$  probability of link good
- Prob[network good] = Prob[all good or (one failed and 4 good)] = Prob[all good] + Prob[one failed and 4 good] =  $q^5 + 5p q^4$

$$\text{Prob[all good]}=q^5 \quad \text{Prob[one failed and 4 good]} = \sum_{j=1}^5 \text{Prob[link } j \text{ failed and all other links good]} = 5pq^4$$

- Prob[network failure] =  $1 - q^5 - 5p q^4$
- Reliability =  $q^5 + 5p q^4$

Switching, impairments, metrics ...

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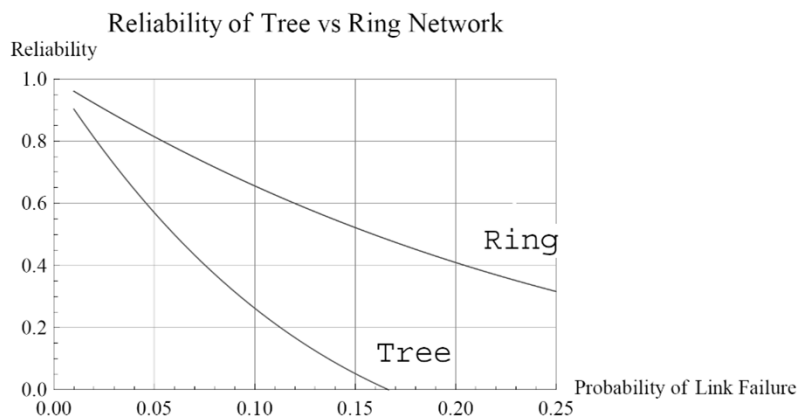
## Network Performance Criteria: Reliability

- Expanding  $\text{Prob}[\text{network failure}] = 10p^2q^3 + 10p^3q^2 + 5p^4q + p^5$
- The dominant term (assuming  $p$  small) is  $10p^2q^3$
- Reliability =  $1 - 10p^2q^3$

Network failure probability

	Tree	Ring
$p$	$4p$	$10p^2q^3$
0.01	0.04	0.00097
0.001	0.004	$10^{-5}$
$10^{-5}$	$4 \times 10^{-5}$	$10^{-9}$
$10^{-7}$	$4 \times 10^{-7}$	$10^{-13}$

## Network Performance Criteria: Reliability



## Other Metrics

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- Call/Session Blocking Probability
- Fairness,
  - N flows
  - Allocate  $X_i$  resources to flow  $i$
  - Jain's Fairness Index (JFI)

$$JFI = \frac{(\sum_{i=1}^N X_i)^2}{N \cdot \sum_{i=1}^N (X_i^2)}$$

- Security

## CoS vs QoS

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- Class of Service,
  - Provides for priority ordering of packet transmission
  - No guarantees of delay or packet loss
  - Example: Video/voice packets are transmitted before "Best Effort(BE)" packets
- Quality of Service
  - Reserve resources for flow
  - Provides statistical performance guarantee
  - Example: 95% of all packets receive a delay of less than 50ms.

# Application Service Requirements

## Data loss

- ❑ some apps (e.g., audio) can tolerate some loss
- ❑ other apps (e.g., file transfer, telnet) require 100% reliable data transfer

## Timing

- ❑ some apps (e.g., Internet telephony, interactive games) require low delay to be "effective"

## Throughput

- ❑ some apps (e.g., multimedia) require minimum amount of throughput to be "effective"
- ❑ other apps ("elastic apps") make use of whatever throughput they get

Modified from *Computer Networking: A Top Down Approach Featuring the Internet*, 4th edition, Jim Kurose, Keith Ross, Addison-Wesley, Copyright 1996-2002, J.F. Kurose and K.W. Ross, All Rights Reserved

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## Service requirements: common apps

<u>application</u>	<u>data loss</u>	<u>throughput</u>	<u>time sensitive?</u>
file transfer/download	no loss	elastic	no
e-mail	no loss	elastic	no
Web documents	no loss	elastic	no
real-time audio/video	loss-tolerant	audio: 5Kbps-1Mbps video: 10Kbps-5Mbps	yes, 10's msec
streaming audio/video	loss-tolerant	same as above	yes, few secs
interactive games	loss-tolerant	Kbps+	yes, 10's msec
text messaging	no loss	elastic	yes and no?

Elastic applications make use of available throughput. Elastic services support this applications.

Modified from: 8th edition Jim Kurose, Keith Ross Pearson, 2020

# Application Requirements

Traffic type	Bit rate	Loss rate	Delay	Jitter
Voice	Low	Medium	High	High
E-commerce	Low	High	High	Low
Transaction	Low	High	High	Low
Email	Low	High	Low	Low
Telnet	Low	High	Medium	Low
Browsing	Medium	High	High	Low
File transfer	High	Medium	Low	Low
Video conferencing	High	Medium	High	High
PnP control message	Low	High	Medium	Low

Jitter is the variation in the delay of consecutive packets

From: [https://www.researchgate.net/figure/Applications-QoS-requirements\\_tbl1\\_41950163](https://www.researchgate.net/figure/Applications-QoS-requirements_tbl1_41950163)

## Network Performance Criteria: Example

TABLE II  
STANDARDIZED QoS CLASS IDENTIFIERS FOR LTE

QCI	Resource Type	Priority	Packet Delay Budget [ms]	Packet Loss Rate	Example services
1	GBR	2	100	$10^{-2}$	Conversational voice
2	GBR	6	150	$10^{-3}$	Conversational video (live streaming)
3	GBR	5	300	$10^{-6}$	Non-Conversational video (buffered streaming)
4	GBR	7	50	$10^{-3}$	Real time gaming
5	non-GBR	9	100	$10^{-6}$	IMS signaling
6	non-GBR	3	100	$10^{-3}$	Voice, video (live streaming), interactive gaming
7	non-GBR	4	300	$10^{-6}$	Video (buffered streaming)
8	non-GBR	8	300	$10^{-6}$	TCP based (e.g., WWW, e-mail), chat, FTP, P2P file sharing
9	non-GBR	1	300	$10^{-6}$	

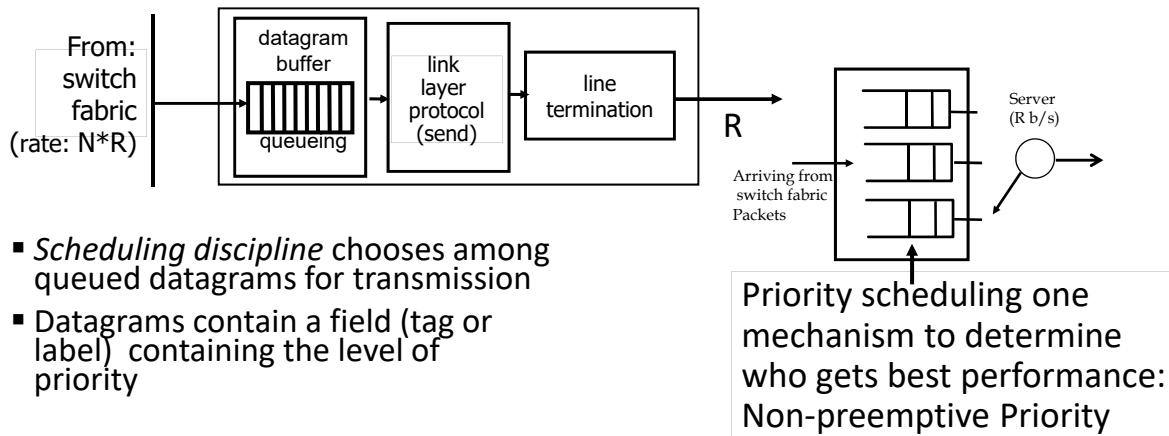
The larger the value of the Priority value the higher priority (more important) the packet

QCI= Quality-of-Service Class Identifier

LTE= Long Term Evolution, aka 4G

GBR = Guaranteed Bit Rate IMS = IP Multimedia Core Network Subsystem

## Implementing Class of Service: Non-preemptive Priority



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## Implementing Class of Service: Non-preemptive Priority

**Packet Priority Assignment:** Each incoming packet has a field (tag or label) containing the level of priority. Here the larger the value of the label the higher priority (more important) the packet.

**Priority Queues:** The router maintains multiple priority queues, each corresponding to a different priority level. Packets are placed into these queues based on their assigned priorities.

**Packet Selection:** The router selects the packet from the highest-priority queue that is ready to be transmitted. Non-preemptive priority means that once a packet is chosen for transmission the packet completes transmission.

**Complete Transmission:** The selected packet is transmitted without interruption. Only when the packet is fully sent will the router consider packets other for transmission.

**Continue Scheduling:** After transmitting a packet, the router selects the highest-priority packet that is ready to be transmitted.

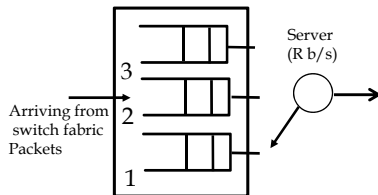
# Implementing Class of Service: Non-preemptive Priority

- Problems with non-preemptive priority
  - Lower priority packets may never get to be sent, blocked by higher priority packets
  - Packets within a priority class are served FIFO, therefore there are no delay/loss guarantees even for higher priority packets.

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# Implementing Class of Service: Non-preemptive Priority

- Example: Traffic going to output port.
  - Class 3 (highest priority, e.g., network management packets.)
    - 100 bits/packet at 1000 packets/sec = 0.1 Mb/s
  - Class 2 (Medium priority, e.g., web packets)
    - 45000 bits/packet at 1000 packets/sec = 45 Mb/s
  - Class 1 (Low priority, e.g., best effort)
    - 3500 bits/packet at 1000 packets/sec = 3.5 Mb/s



- Case 1: High load
  - $R = 50 \text{ Mb/s}$
  - Total Load  $= (0.1 + 45 + 3.5) / 50 = 97.2\%$
  - Class 3 packet may have to wait  $45000 / (50 \text{ Mb/s}) = 0.9 \text{ ms}$  for one class 1 packet to complete before on opportunity to transmit
  - All classes see finite delays (large)
- Case 2: Over loaded
  - $R = 47 \text{ Mb/s}$
  - Total load  $> 1$
  - Class 3 + Class 2 Load =  $\sim 96\%$  & see finite delays
  - Class 1 packets see an over loaded system and average delay  $\rightarrow \infty$
- Case 3: Low Load
  - $R = 1 \text{ Gb/s}$  Total Load  $= (0.1 + 45 + 3.5) / 1000 = \sim 5\%$
  - Average delay  $\approx$  clocking time, i.e., queues likely empty

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## Network Design Problem

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- Goal
  - Given
    - QoS requirement, e.g.,
      - Average delay
      - Loss probability
    - Characterization of the traffic: the input to the network
      - Common traffic characteristics
        - Average interarrival time (arrival rate)
        - Average holding time (message length)
  - Design the system

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## Network Performance Evaluation

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- Solution methodologies:
  - Mathematical analysis
    - Model this type of process as a Queueing System → good for initial design
    - Provide insights into protocol operation and performance
  - Simulation techniques → good for more detailed analysis

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# Performance

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## □ Traffic modeling

- Describes the nature of what is transported over communications networks.
- Understanding traffic can be used to improve network performance
- Traffic is random
  - Time between packet arrivals, interarrival time,  $T_a$  is now a random variable
  - Average rate of packet arrivals =  $\lambda$ , e.g., in packets/sec
  - Packet length,  $L$ , is now a random variable
    - $E[L]$  = Expected value of the length (mean or average), e.g., in bits/packet
    - Clocking time (Holding time =  $L/R=T_H$ ) is now a random variable
      - $E[L]/R$ ,
      - Example,  $E[L]=1000$  bits,  $R=1\text{Mb/s}$  then average holding time = 1ms

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# Traffic Characterization

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- Customers request information
- Rate of requests =  $\lambda$  requests/sec
  - Calls/sec
  - Packets/sec
  - mp3's/hour
- The volume of information requested
  - Length of the phone call (sec/call)
  - Length of movie (Bytes)
  - Size of picture (Bytes)

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# Traffic Characterization

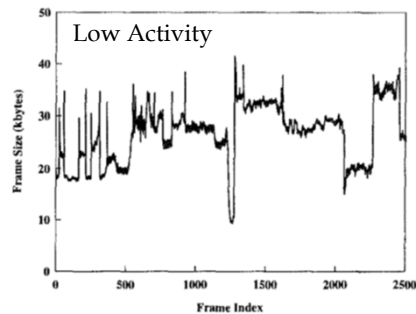
- Traffic characterization describes the user demands for network resources
  - How often a customer:
    - Requests a web page
    - Down loads an MP3
    - Makes a phone call
  - Size/length (how long you hold network resources)
    - Web page
    - Song
    - Phone call

See: V.S. Frost & B. Melamed, Traffic modeling for telecommunications networks, IEEE Communications Magazine, Volume: 32 Issue: 3, March 1994

# Sample Realization of an Traffic Process

Message number	1	2	3	4	5	6	7	8	9	10	11	12
Interarrival time between $i+1$ and $i$ message (seconds)	2	1	3	1	1	4	2	5	1	4	2	--
Length of $i^{\text{th}}$ message (seconds)	1	3	6	2	1	1	4	2	5	1	1	3

Arrival Events & Lengths →



A TES-based model for compressed "Star Wars" video, B. Melamed, D. Pendarakis, 1994 IEEE GLOBECOM. Communications: Communications Theory Mini-Conference Record

## Traffic:

### General Characteristics

---

- Highly variable
- Likely to change as new services and applications evolve.
- Highly bursty, where one definition of burstyness is:

$$\text{Burstyness} = \frac{\text{Peak rate}}{\text{Average rate}}$$

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## Traffic:

### General Characteristics

---

Example: During a remote login connection over a 19.2kb/s modem a user types at a rate of 1 symbol/sec or 8 bits/sec and then transfers a 100 kbyte file. Assume the total holding time of the connection is 10 min.

What is the burstyness of this data session?

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## Traffic:

### General Characteristics

The time to transfer the file is  
 $(800,000 \text{ bits}) / (19,200 \text{ b/s}) = 41 \text{ sec.}$   
So for 600 - 41sec = **559 sec.**  
the data rate is 8 bits/sec or  
4,472 bits were transferred in 559 sec.  
Thus in 600 sec. 4,472 + 800,000 bits were transferred,  
yielding a average rate of:  
 $804,472 \text{ bits} / 600 \text{ sec} = \mathbf{1,340 \text{ bits/sec.}}$   
The peak rate was 19.2 Kb/s so the burstyness for this  
data session was:

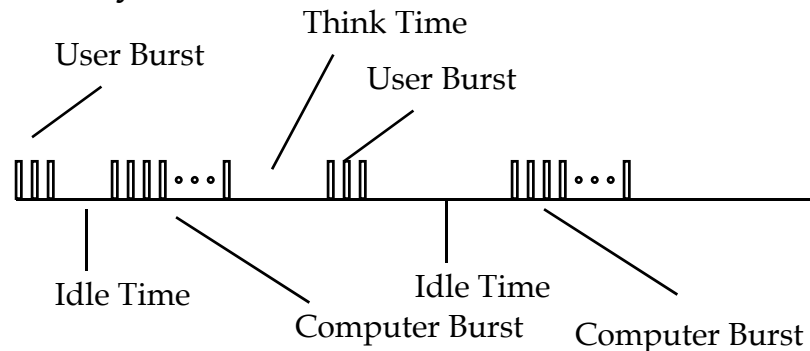
$$19,200 / 1,340 = 14.3$$

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## Traffic:

### General Characteristics

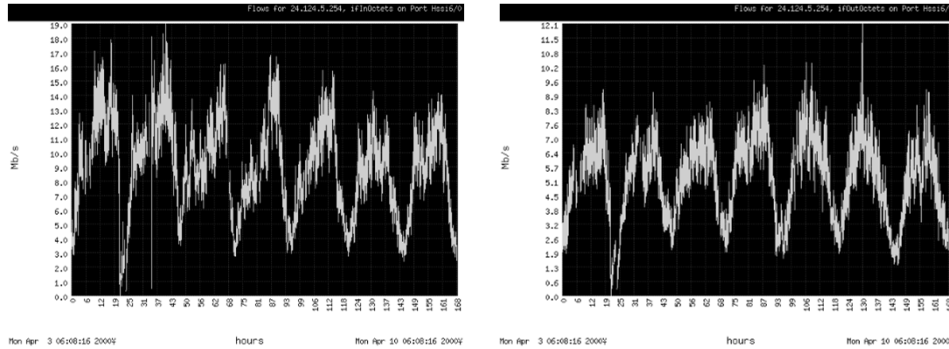
#### Asymmetric Nature of Interactive Traffic



This Asymmetric property has lead to asymmetric services

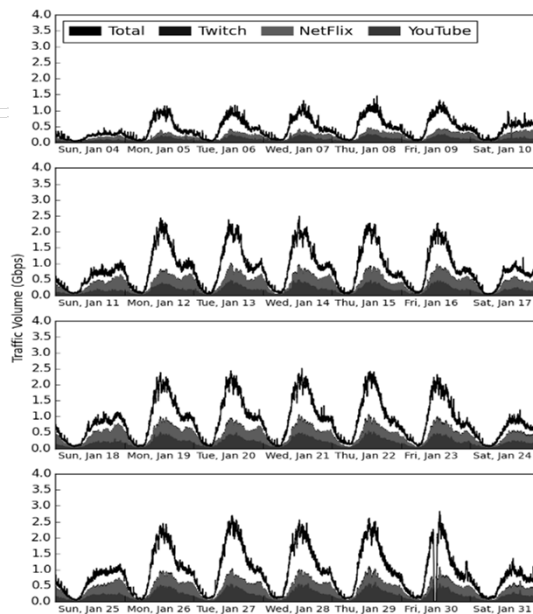
54

## Time of Day Variations



From the Internet into Datavision  
 Mean = 8.876 Mb/s.  
 Maximum = 18.952 Mb/s

From Datavision out to the Internet  
 Mean = 5.133 Mb/s.  
 Maximum = 12.093 Mb/s



## Video Traffic

- January 2015
- Top line (Total) is HTTP+HTTPS
- Red is (HTTPS) YouTube
- Green is NetFlix
- Blue is Twitch

## In General Traffic

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- Very bursty
- Problems with traffic modeling
  - Rapidly evolving applications
  - Complex network interactions

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## Packet Voice (applies to packet video)

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- Packet voice/video looks like a steady flow or Constant Bit Rate (CBR) traffic
- However, voice/video can be Variable Bit Rate or VBR
  - *"silence detection"*
  - *Variable rate coding*
- Problem: After going through the network the packets will not arrive equally spaced in time. Thus playback of packet voice must deal with variable network delays

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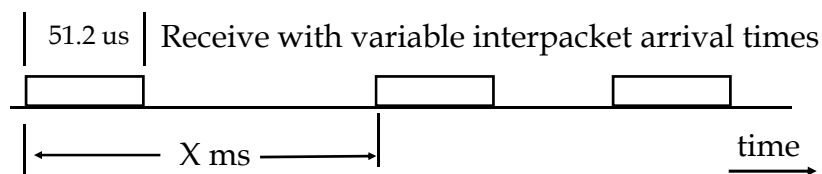
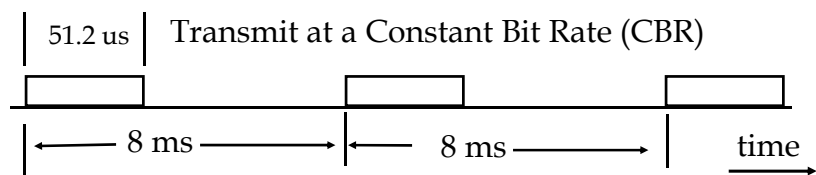
## Packet Voice (applies to packet video)

### □ Example: Parameters for a packet voice system

- 1 source
- Sample rate = 8000 samples/sec (ITU G.711)
- 8 bits/sample (1 byte/sample)
- **8 ms of voice/packet** ← *Critical parameter*
- Packet size (bytes/packet) =  $(8\text{ms/packet}) \cdot (8000\text{ bytes/sec}) = 64\text{ Bytes}$   
[assuming no overhead bytes]
- Link rate = 10 Mb/s
- Clocking time/packet  
(or holding time/packet or service time/packet aka, serialization time)  
=  $(64\text{bytes/packet}) \cdot (8\text{bits/byte}) / (10\text{ Mb/s}) = 51.2\text{us}$

59

## Voice Traffic: Packet Voice



X not equal 8ms because of random network delays  
If X is too big packet may arrive too late for play out

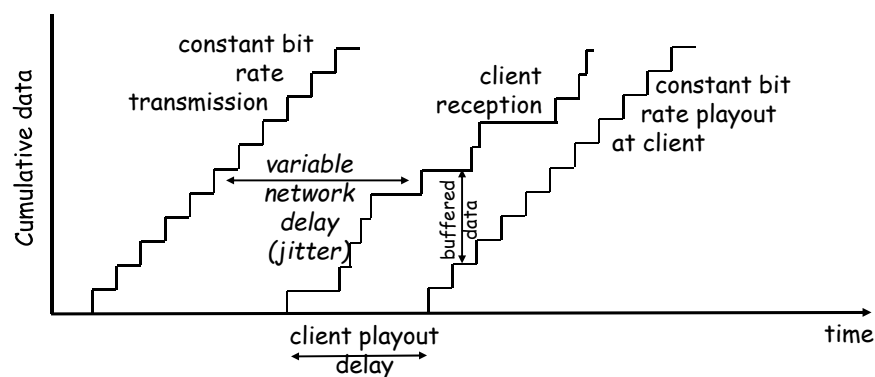
60

## Voice Traffic: Packet Voice

- Assume network delay is uniformly distributed between [25 ms, 75 ms]
  - Same as having a fixed propagation delay of 25 ms with a random network delay uniformly distributed between [0 ms, 50 ms]
- Note receiver will run out of bytes to playout after 8 ms.
- Solution:
  - Jitter Buffer Memory to hold 50 ms of the voice signal (or 8 packets or 2.8 Kbits)
  - Worst case, receiver will run out of data just as a new packet arrives

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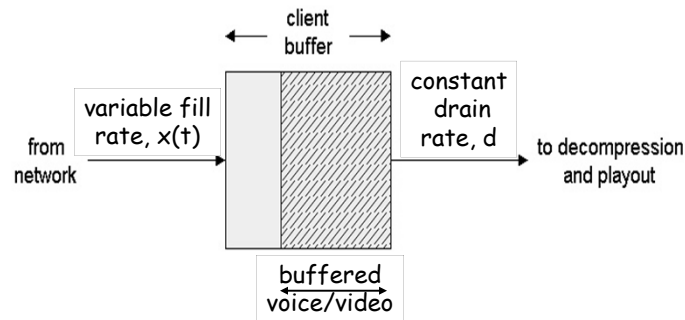
## Delay Jitter



- consider end-to-end delays of two consecutive packets: difference is random (transmission time difference)

*Modified from: Computer Networking: A Top Down Approach, 4<sup>th</sup> edition. Jim Kurose, Keith Ross Addison-Wesley, July 2007.*

## Streaming Multimedia: Client Buffering



- client-side buffering, playout delay compensate for network-added delay, delay jitter

*Modified from: Computer Networking: A Top Down Approach, 4<sup>th</sup> edition. Jim Kurose, Keith Ross Addison-Wesley, July 2007.*

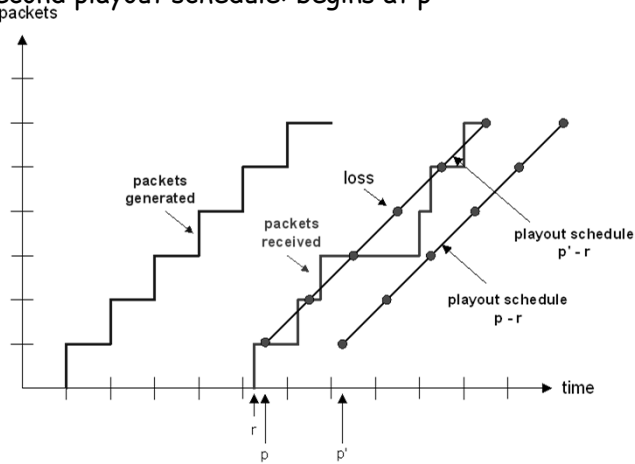
## Voice Traffic: Packet Voice

- New problem: networks delays are unknown and maybe unbounded
- A voice packet may arrive at 85 ms and be *too late* to be played back
  - Late packets are dropped
  - Last packet may be played out in dead time
- Packet voice (video) schemes must be able to deal with variable delay and packet loss  
(Should voice/video packets be retransmitted?)



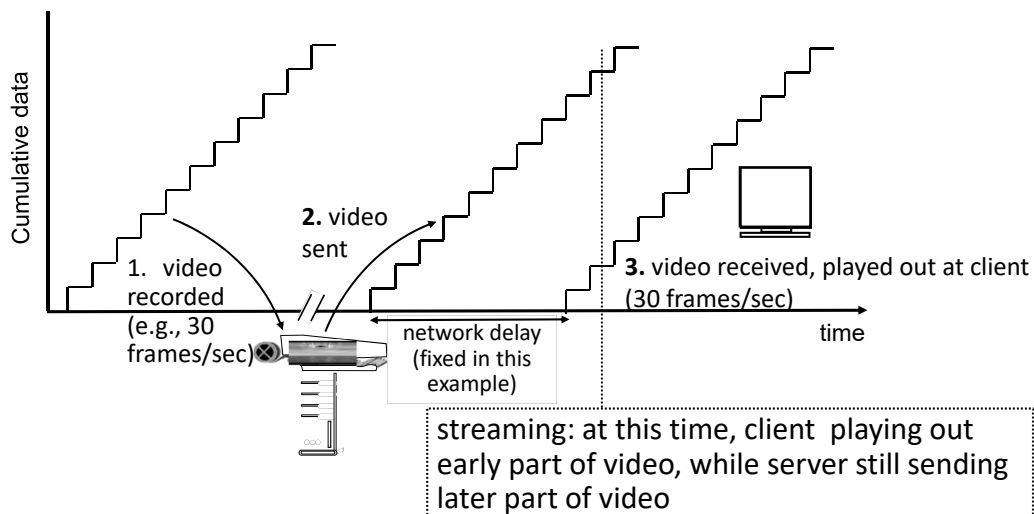
## Fixed Playout Delay

- sender generates packets every 20 msec during talk spurt.
- first packet received at time  $r$
- first playout schedule: begins at  $p$
- second playout schedule: begins at  $p'$



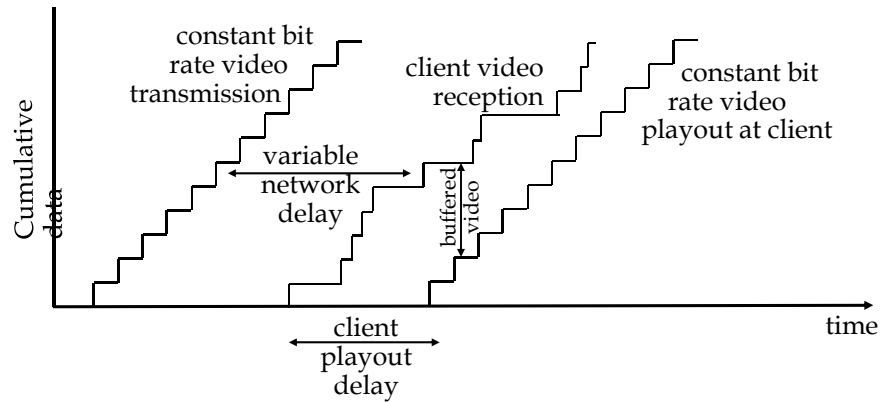
Modified from: *Computer Networking: A Top Down Approach*, 4<sup>th</sup> edition. Jim Kurose, Keith Ross Addison-Wesley, July 2007.

## Streaming stored video



Modified from: 8<sup>th</sup> edition Jim Kurose, Keith Ross Pearson, 2020

## Streaming stored video: playout buffering



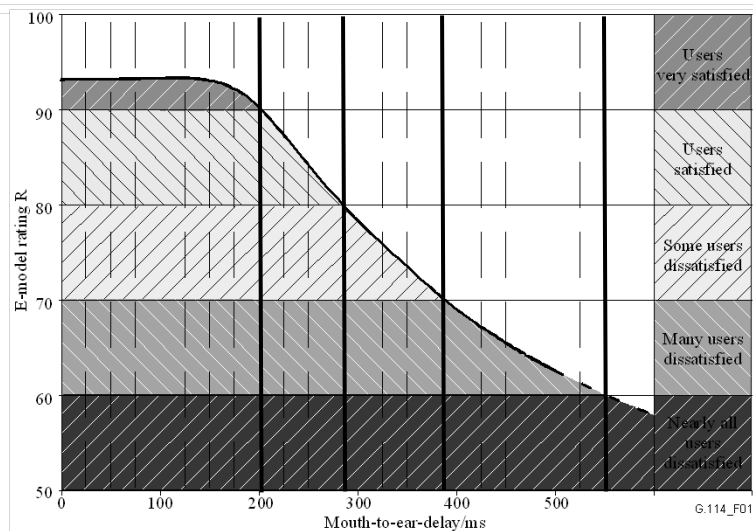
- *client-side buffering and playout delay*: compensate for network-added delay, delay jitter

Modified from: 8<sup>th</sup> edition Jim Kurose, Keith Ross Pearson, 2020

## Voice over IP (VoIP) Quality

The mouth-to-ear delay is the time taken from when a user begins to speak until when the listener actually hears the speech. This one-way latency is known as mouth-to-ear delay.

The E-model (ITU-T Rec. G. 107) is a transmission planning tool that provides a prediction of the expected voice quality, as perceived by a typical telephone user

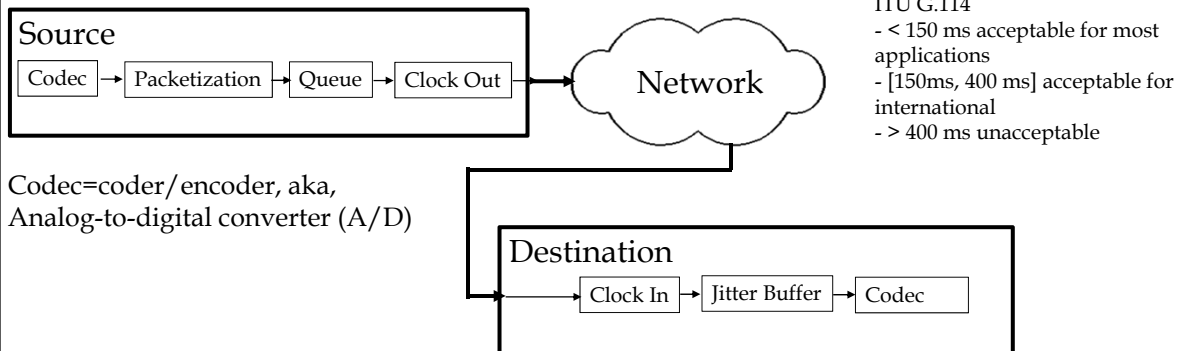


ITU-T Recommendation G.114-One-way transmission time, May 2003

# VoIP- Delay budget

## Factors in End to End Delay

Assumption: maximum delay from mouth-to-ear needs to be on the order of 200 -300 ms

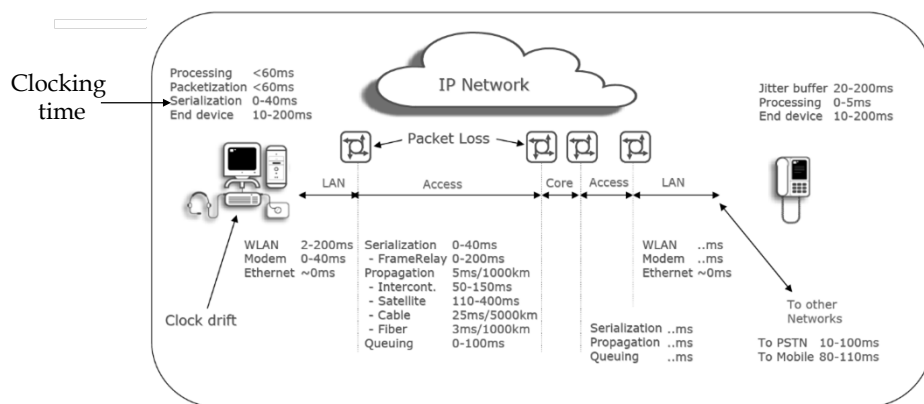


ITU G.114  
 - < 150 ms acceptable for most applications  
 - [150ms, 400 ms] acceptable for international  
 - > 400 ms unacceptable

Codec=coder/encoder, aka, Analog-to-digital converter (A/D)

Clock Out, aka, serialization, modeled by the "server"

# Delay & Packet Loss Sources



PSTN=Public Switched Telephone Network

# VoIP- Delay budget

## Factors in End-to-End Delay

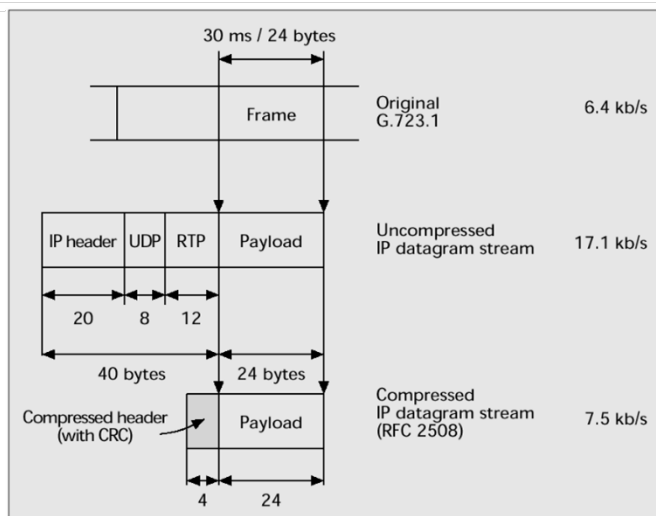
- Example: Delay Budget (depends on assumptions)
  - Formation of VoIP packet at TX ~ 30 ms  
20ms of voice/packet is default for Cisco 7960 router
  - Other VoIP packet processing ~70 ms  
(see: <http://www.rmav.arauc.br/pdf/voip.pdf>)
  - Propagation ~10 ms
  - Network Delays ~10 ms
  - Extraction of VoIP packet at Receiver ~30 ms
  - Jitter Buffer ~ 100 ms  
Compensates for variable network delay
  - Total 250 ms

- Possible trade-offs:
  - Jitter Buffer vs voice packet loss
  - VoIP packet size vs length of jitter buffer

For examples see: <https://www.cisco.com/c/en/us/support/docs/voice/voice-quality/5125-delay-details.html#packetizationdelay>

Ref: [http://www.lightreading.com/document.asp?site=lightreading&doc\\_id=53864&page\\_number=6](http://www.lightreading.com/document.asp?site=lightreading&doc_id=53864&page_number=6)

## Voice Traffic: Packet Voice



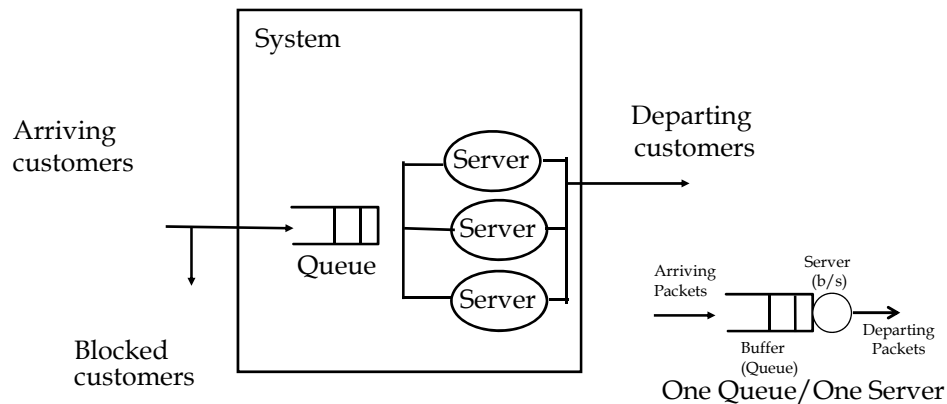
G.723.1 is a voice coding standard, linear prediction compression algorithm

From: Performance Evaluation of the Architecture for End-to-End Quality-of-Service Provisioning, Katsuyoshi Iida, Kenji Kawahara, Tetsuya Takine, and Yuji Oie, IEEE Communications Magazine, April 2000

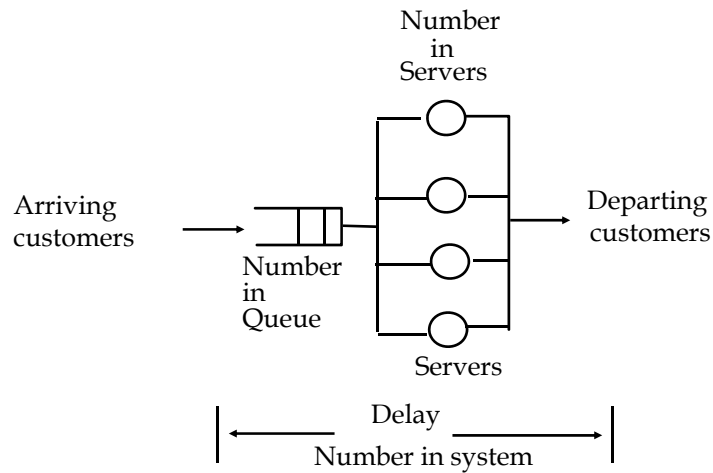
# Network Design Problem

- Goal
  - Given
    - QoS requirements, e.g.,
      - Average delay
      - Loss probability
    - Characterization of the traffic, e.g.,
      - Average interarrival time (arrival rate)
      - Average holding time (message length)
  - Design the system, e.g., determine link capacity and system size
  - Three systems will be studied:
    - Ideal (infinite buffer) router output port, e.g., determine link capacity  
System 1 →  $M/M/1$  ( $M/M/1/\infty/\infty$ )
    - Real router output port (finite buffer), e.g., determine link capacity and buffer size  
System 2 →  $M/M/1/S$  ( $M/M/1/S/\infty$ )
    - Circuit switch, e.g., determine the # lines  
System 3 →  $M/M/S/S$  ( $M/M/S/S/\infty$ )

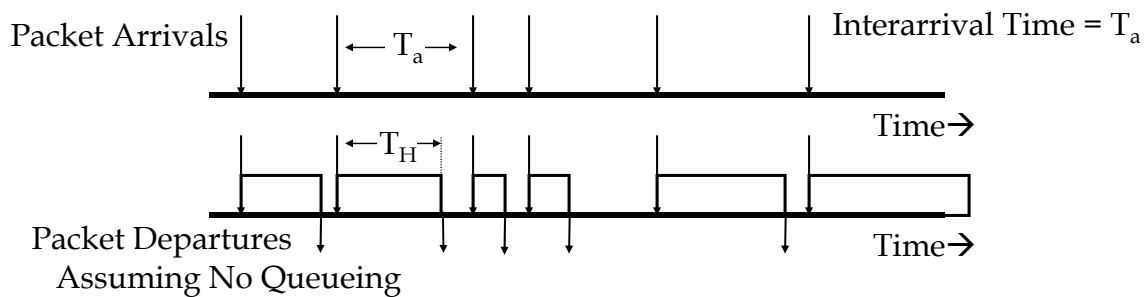
# Network Performance Evaluation: Elements of a Queueing System



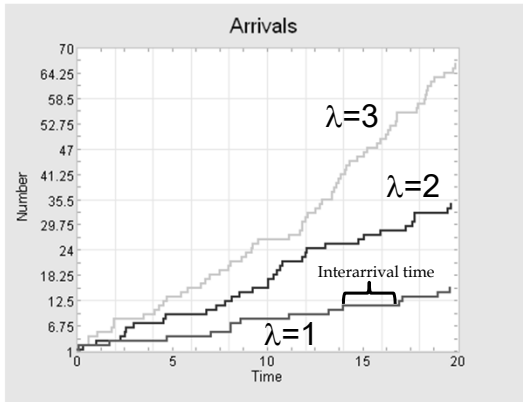
## Network Performance Evaluation: Elements of a Queueing System



## Traffic: Arrivals & Departures

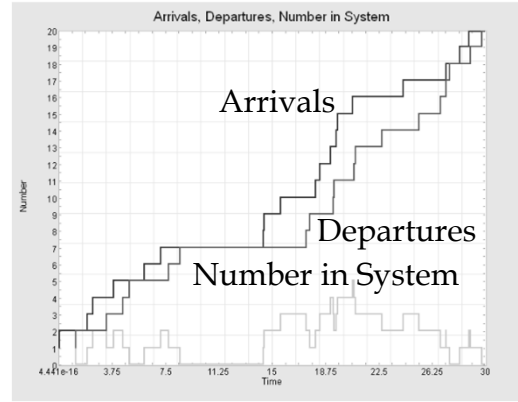


$T_H$  = Holding, aka Clocking, aka Service time =  $L/R_{out}$   
 Average Service time =  $E[L]/R_{out}$   $E[\ ]$  = Expected value



Arrival Process

$\lambda$  = Arrival rate (Packets/sec)



Arrival-Departure Process

(Birth - Death Process)

## Network Performance Evaluation:

### Assumptions and Definitions

- Packet interarrival times  $T_a$  are exponentially distributed - Markov Process
  - Arrival Rate (packets/sec) =  $\lambda$   $P[T_a < t] = 1 - e^{-\lambda t}$
- Clocking, (Service or Holding) times  $T_H$  are exponentially distributed - Markov Process
  - Packet length =  $L$  = packet length in bits
  - Average Service (Holding time) =  $E[T_H] = E[L]/R_{out}$  where  $R_{out}$  = link capacity in b/s
  - Service rate (packets/sec) = Departure rate (packets/sec) =  $\mu = 1/E[T_H] = R_{out}/E[L]$
- Average input rate (b/s)  $R_{in} = \lambda E[L]$  bits/sec
- Average departure rate (b/s)  $R_{out} = \mu E[L]$  bits/sec
- Traffic intensity (load) =
  - $\rho = R_{in}/R_{out} = \lambda E[T_H] = (\lambda E[L])/R_{out} = \lambda/\mu = \text{Arrival rate}/\text{service rate (units Erlang)}$

## Show animated example

---

- Arrival - Departure Process

## Example

---

- Average packet length = 1000 bytes/packet
- Output link rate =  $R_{out} = 50 \text{ Mb/s}$
- Arrival rate =  $\lambda = 4000 \text{ packets/sec}$ 
  - $R_{in} = \lambda E[L] = (4000 \text{ packets/sec}) * (8 * 1000 \text{ bits/packet}) = 32 \text{ Mb/s}$
  - Load =  $\rho = 32/50 = .64$
  - Service rate  $\mu = (50 \text{ Mb/s}) / (8 * 1000 \text{ bits/packet}) = 6250 \text{ packets/sec}$
  - Load =  $\rho = \lambda / \mu = 4000 / 6250 = R_{in} / R_{out} = 0.64$




## Network Performance Evaluation: Queueing System Notation (Kendall's notation)


- $A / b / m / K / L$
- $A$  = type of arrival process: **M=Markov Process**
  - Time between arrivals has an exponential pdf
- $b$  = type of service process: **M=Markov Process**
  - Service time between arrivals has an exponential pdf
- $m$  = number of servers
- $K$  = maximum number of elements allowed in the system = system size (if  $K$  missing then  $\infty$ )
- $L$  = population size (if  $L$  missing then  $\infty$ )

## Network Performance Evaluation: Specific cases

- Three types of systems considered in this class

➤ One server (Stat Mux) → Router output port

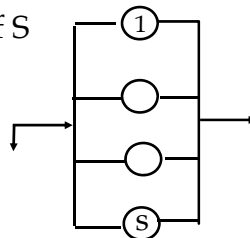
- Infinite memory-M/M/1 → 

- Finite memory (if full then drop packet) - M/M/1/S 

➤  $S$  servers and a system size of  $S$

(if full then drop) - M/M/S/S

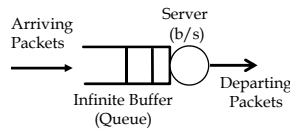
No buffer/queue



## Network Performance Evaluation: Summary of results for specific cases

### □ M/M/1

- One Server
- Infinite system size
- Link Rate=  $R_{out}(b/s)$
- $L$  = Packet Length (bits): pdf ~ exponential;  
Average packet length= $E[L]$
- Packet Arrival rate =  $\lambda$  (packets/sec)
- $T_a$  = interarrival times: pdf ~ exponential;
- Service rate= $\mu= R_{out}/E[L]$  (packets/sec)
- $E[T_H]=E[L]/R_{out}=1/\mu$
- Load  $=\rho = R_{in}/R_{out} = \lambda E[T_H] = \lambda E[L]/R_{out} = \lambda / \mu$



Probability of  $k$  in system= $P[K=k] = \rho^k(1-\rho)$   
 Probability of system busy = utilization=  $\rho$   
 Probability of system empty =  $1-\rho$

Average Number in System =  

$$E[K] = \frac{\rho}{1-\rho}$$

Variance of Number in System =  

$$\text{Var}[K] = \frac{\rho}{(1-\rho)^2}$$

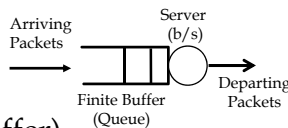
Average Delay = $E[D]=$   

$$\frac{E[T_H]}{1-\rho} = \frac{E[L]/R_{out}}{1-\rho} = \frac{1}{\mu-\lambda}$$

## Network Performance Evaluation: Summary of results for specific cases

### □ M/M/1/S

- One Server
- System size =  $S$  (server+buffer)
- Rate= $R_{out}(b/s)$
- Packet arrival rate =  $\lambda$  (packets/sec)
- $L$  = Packet Length (bits): pdf ~ exponential;  $E[L]$
- $T_a$  = interarrival times: pdf ~ exponential;
- $E[T_H]=E[L]/R_{out}$
- Load  $=\rho = R_{in}/R_{out} = \lambda E[T_H] = \lambda E[L]/R_{out}$



$$P[K = k] = \frac{(1-\rho)\rho^k}{1-\rho^{S+1}} \text{ for } k \leq S$$
  

$$P[K = k] = 0 \text{ for } k > S$$

$$P_{Blocking} = P[K = S] = \frac{(1-\rho)\rho^S}{1-\rho^{S+1}}$$

Table to be provided on test and Excel spreadsheet provided on class web site see

<http://www.ittc.ku.edu/~frost/EECS563/M-M-1-K-Blocking%20cal.xls>

## Network Performance Evaluation: Summary of results for specific cases

### □ M/M/S/S (Erlang B)

- S= Servers
- S=System size
- Arrival rate =  $\lambda$  (items/sec)
- L = Item Length (sec): pdf ~ exponential;  
E[L]
- $T_a$  = interarrival times: pdf ~ exponential;
- Load =  $\rho = \lambda E[L] \rightarrow$  units Erlang

$$P[K = k] = \frac{\rho^k}{k!} / \sum_{n=0}^S \frac{\rho^n}{n!}$$

$$P[K = k] = 0 \text{ for } k > S$$

$$P_{\text{Blocking}} = P[K = S] = \frac{\rho^S / S!}{\sum_{n=0}^S \frac{\rho^n}{n!}}$$

Erlang B blocking Formula

Tabulated and there are web calculators see:  
<http://www.erlang.com/calculator/index.htm>  
[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/erlang-table.pdf](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/erlang-table.pdf)

Network Performance...

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## Network Performance Evaluation: Summary of results for specific cases

### □ M/M/1

Probability of k in system =  $P[K=k] = \rho^k(1-\rho)$   
 Probability of system busy = utilization =  $\rho$   
 Probability of system empty =  $1-\rho$

Average Number in System =

$$E[K] = \frac{\rho}{1-\rho}$$

Variance of Number in System =

$$\text{Var}[K] = \frac{\rho}{(1-\rho)^2}$$

Average Delay =  $E[D]$  =

$$\frac{E[T_H]}{1-\rho} = \frac{E[L]/R_{out}}{1-\rho} = \frac{1}{\mu - \lambda}$$

$$\text{Load} = \rho = R_{in} / R_{out} = \lambda E[T_H] = \lambda E[L] / R_{out} = \lambda / \mu$$

### □ M/M/1/S

$$P[K = k] = \frac{(1-\rho)\rho^k}{1-\rho^{S+1}} \text{ for } k \leq S$$

$$P[K = k] = 0 \text{ for } k > S$$

$$P_{\text{Blocking}} = P[K = S] = \frac{(1-\rho)\rho^S}{1-\rho^{S+1}}$$

Table to be provided on test and Excel spreadsheet provided on class web site see [http://www.ittc.ku.edu/~frost/EECS\\_563/M-M-1-K-Blocking%20cal.xls](http://www.ittc.ku.edu/~frost/EECS_563/M-M-1-K-Blocking%20cal.xls)

### □ M/M/S/S

$$P[K = k] = \frac{\rho^k}{k!} / \sum_{n=0}^S \frac{\rho^n}{n!}$$

$$P[K = k] = 0 \text{ for } k > S$$

$$P_{\text{Blocking}} = P[K = S] = \frac{\rho^S / S!}{\sum_{n=0}^S \frac{\rho^n}{n!}}$$

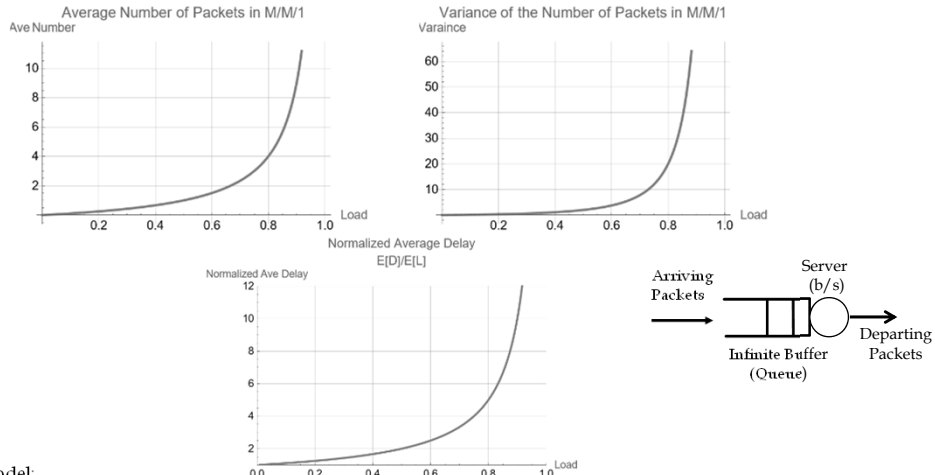
Erlang B blocking Formula

Tabulated and there are web calculators see:  
<http://www.erlang.com/calculator/index.htm>  
[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/erlang-table.pdf](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/erlang-table.pdf)

Network Performance...

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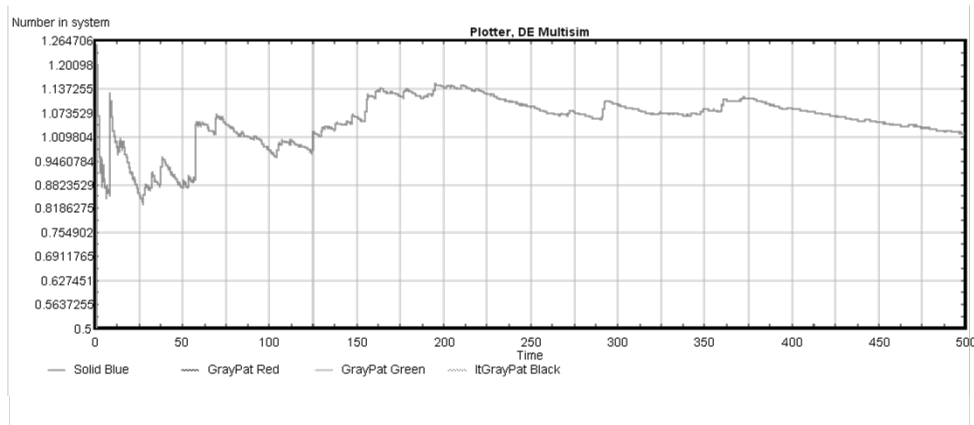
# Delay Analysis for M/M/1



Show Extend model:  
[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/Extend\\_Models\\_2019-v10/Stat Mux Delay and Number model-ES10.mox](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/Extend_Models_2019-v10/Stat_Mux_Delay_and_Number_model-ES10.mox)

Network Performance...

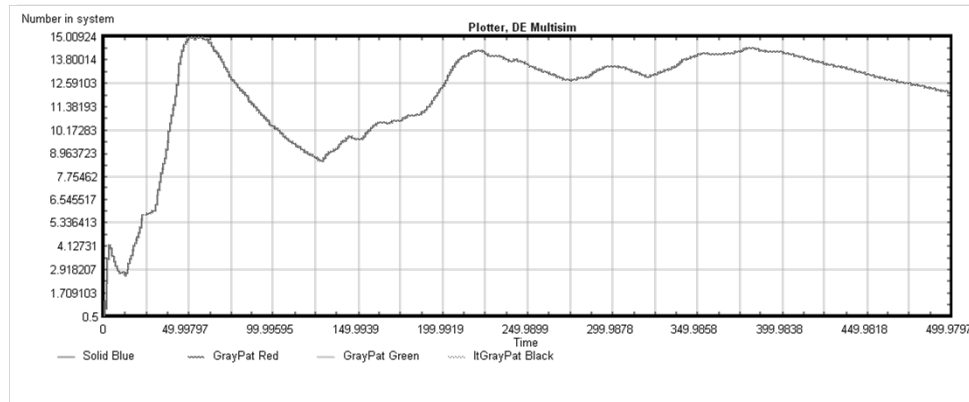
# Network Performance Evaluation: M/M/1



$$\lambda=500 \text{ \& } E[L]= 1000 \text{ bits, } C = 1\text{Mb/s} \rightarrow \text{Load} = 0.5 \quad E[K] = 1$$

Network Performance...

## Network Performance Evaluation: M/M/1

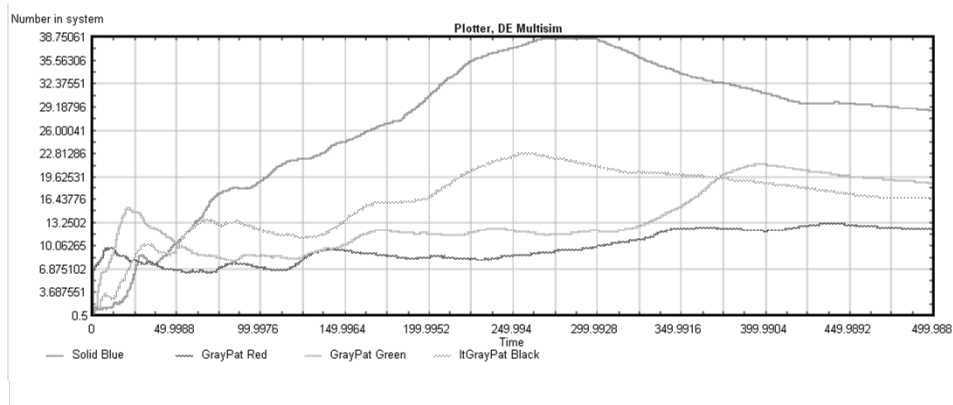


$\lambda=950$  &  $E[L]=1000$  bits,  $C = 1\text{Mb/s} \rightarrow \text{Load} = 0.95$   $E[K] = 19$   
Final simulated value = 12.05

Network Performance...

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## Network Performance Evaluation: M/M/1 Impact of High Load on Variance



Load = 0.95  $E[K] = 19$

Network Performance...

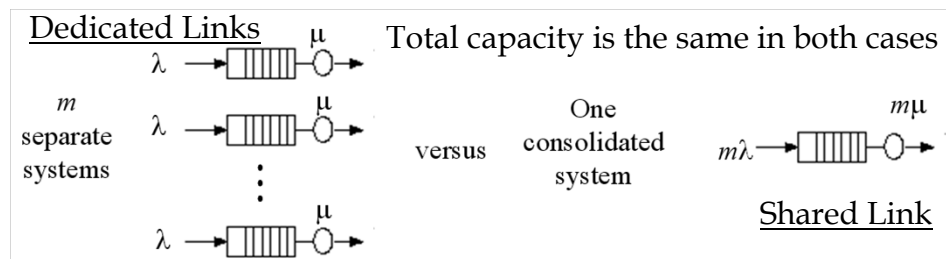
90

## Examples of Delay Analysis for M/M/1

- **Example 1:**  $\lambda=950$ ,  $E[L]= 1000$  bits,  $R_{out} = 1\text{Mb/s}$ 
  - $E[L]/ R_{out} = 1\text{ms} \rightarrow \text{Load}=\rho=0.95$
  - $E[\text{Delay}]= 1\text{ms}/(1-0.95) = 20$  ms
- **Example 2:**  $\lambda=500$ ,  $E[L]= 1000$  bits,  $R_{out} = 1\text{Mb/s}$ 
  - $E[L]/R = 1\text{ms} \rightarrow \text{Load}=\rho=0.5$
  - $E[\text{Delay}]= 1\text{ms}/(1-0.5) = 2$  ms
- **Example 3:**  $\lambda=100$ ,  $E[L]= 1000$  bits,  $R_{out} = 1\text{Mb/s}$ 
  - $E[L]/R = 1\text{ms} \rightarrow \text{Load}=\rho=0.1$
  - $E[\text{Delay}]= 1\text{ms}/(1-0.1) = 1.11$  ms  $\sim E[L]/ R_{out} = 1\text{ms}$
  - At low loads  $\sim$  no queueing and delay = service time

## Network Performance Evaluation: Example

- Which is better?



Example: Traffic:

Assume average packet length=1000 bits/packet

Assume arrival rate =  $\lambda = 50$  packets/sec/system

## Network Performance Evaluation: Example

Example: Traffic:

Assume average packet length=1000 bits/packet

Assume arrival rate =  $\lambda = 50$  packets/sec/system

### □ Dedicated Links

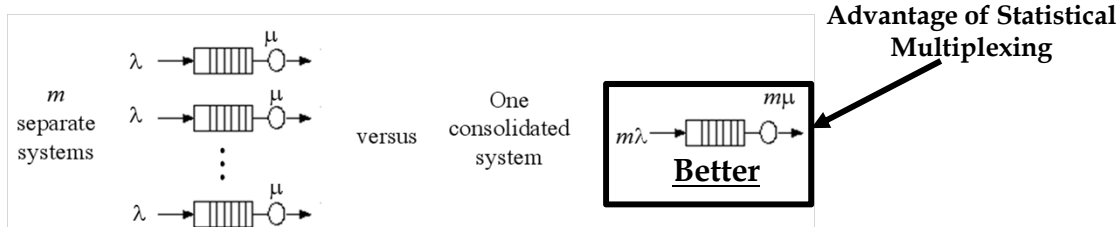
- 10 ( $m=10$ ) systems each with their dedicated 100 kb/s link to a server. (total capacity =  $10 \cdot 100 \text{ kb/s} = 1 \text{ Mb/s}$ )
- Service time =  $1000 \text{ bits} / 100,000 \text{ b/s} = 10 \text{ ms}$
- Traffic intensity=load =  $50 \cdot 1000 / 100,000 = 0.5$
- Average delay =  $10 \text{ ms} / .5 = 20 \text{ ms}$

Network Performance...

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## Network Performance Evaluation: Example-continued

- Shared link: all traffic shares one statistical multiplexer with  $R = 1 \text{ Mb/s}$ 
  - Traffic intensity = 0.5
  - Service time = 1 ms
  - Average delay = 2 ms
- This shows that traffic aggregation improves system performance



Network Performance...

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Do the Design Problem:

Find the link capacity between the stat mux and the server such that the delay is 20 ms.

Example: Assume average packet length=1000 bits/packet  
Assume arrival rate =  $\lambda = 50$  packets/sec/system  
10 systems sharing one link

$$E[D] = 0.02 \text{ sec} = \frac{1}{\mu - \lambda}$$

$$\mu - \lambda = 50$$

$$\lambda = 500 \text{ (packets/sec)}$$

$$\mu = 550 \text{ packets/sec}$$

$$R_{out} = 1000(\text{bits/packet})550(\text{packets/sec}) = 550\text{kb/s}$$

Network Performance...

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## Network Performance Evaluation: Example: Finite buffers- M/M/1/S

□ Find blocking probability for this system:

□ Traffic specification:

- > Average packet length = 1000 bits
- > Arrival rate = 700 packets/sec

□ System Specifications:

- >  $R_{out} = 1 \text{ Mb/s}$
- > System size (buffer + server) = 9 packets

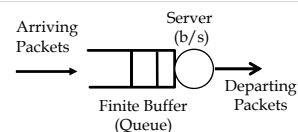
□  $R_{in} = \lambda E[L] = 700\text{kb/s}$

□ Load  $\rho = R_{in} / R_{out} = 0.7$

$$P_{Blocking} = P[K = S] = \frac{(1 - \rho)\rho^S}{1 - \rho^{S+1}}$$

□ Or use [http://www.ittc.ku.edu/~frost/EECS\\_563/M-M-1-K-Blocking%20cal.xls](http://www.ittc.ku.edu/~frost/EECS_563/M-M-1-K-Blocking%20cal.xls)

□ Blocking Probability  $\sim 1.2\%$



Show Extend Model:

[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/Extend\\_Models\\_2019-v10/Stat-Mux-Finite-System-Throughput-Blocking\\_Delay-ES10.mox](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/Extend_Models_2019-v10/Stat-Mux-Finite-System-Throughput-Blocking_Delay-ES10.mox)

Network Performance...

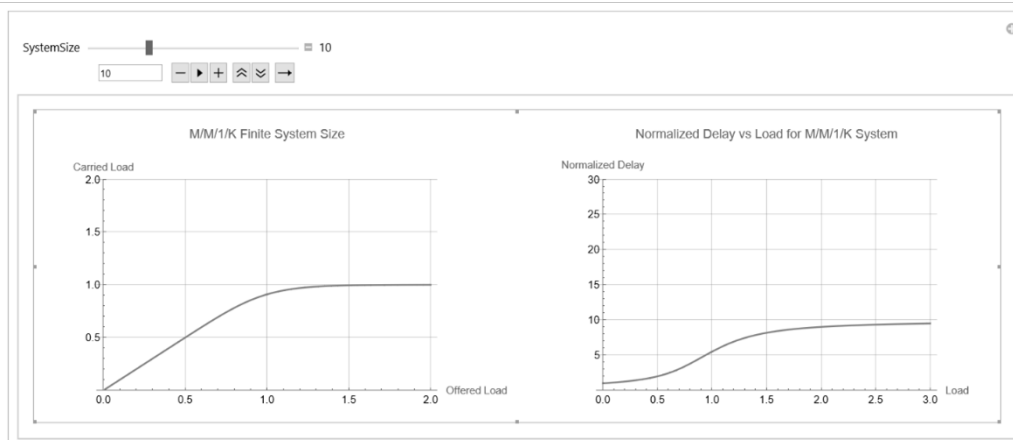
96



## Network Performance Evaluation: Example: Design problem for M/M/1/S

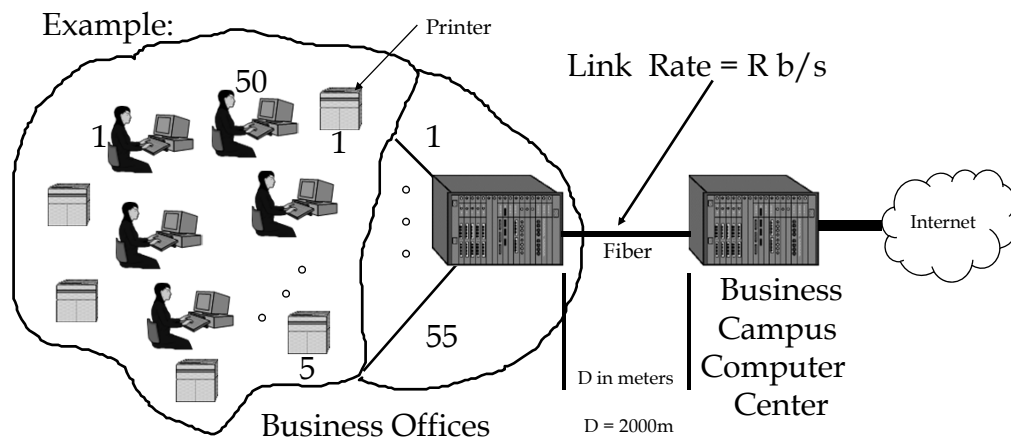
- Find the link rate  $R_{out}$  to achieve a blocking probability of  $\sim 3\%$ .
- Traffic specification:
  - Average packet length = 1000 bits
  - Arrival rate = 900 packets/sec
- System Specifications:
  - System size (buffer + server) = 9 packets
- use [http://www.ittc.ku.edu/~frost/EECS\\_563/M-M-1-K-Blocking%20cal.xls](http://www.ittc.ku.edu/~frost/EECS_563/M-M-1-K-Blocking%20cal.xls)
- From table a load = 0.8 provides a Block Probability  $\sim 3\%$
- Load =  $R_{in} / R_{out} = 0.8$
- $R_{in} = 900 \text{ kb/s}$
- $R_{out} = 1.125 \text{ Mb/s}$
- At this load what is the maximum delay (for packets transmitted)?
- $9 \times 1000 \text{ bits} / 1.125 \text{ Mb/s} = 9 \text{ service times} = 8 \text{ ms}$

## Network Performance Evaluation: Trade-offs- Finite buffers - M/M/1/S



See: [http://www.ittc.ku.edu/~frost/EECS\\_563/M-M-1-K-Delay-and-load.cdf](http://www.ittc.ku.edu/~frost/EECS_563/M-M-1-K-Delay-and-load.cdf)

## Network Performance Evaluation: Design Example



Network Performance...

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## Network Performance Evaluation: Design Example

- Design the system, i.e., find the system size and link rate,  $R_{out}$ , to meet the customer requirements
  - Delay < 100 ms
  - Loss < 10%
- Assume customer traffic:
  - Average packet length = 9000 bytes/packet
  - 55 sources
  - Packets are generated at a rate of 2 per second/source
- Approach (This is results in an over designed system, why?)
  - Find  $R_{out}$  first using only the delay specification, Delay < 100 ms, with the M/M/1 result, i.e., assume infinite systems size to find  $R_{out}$  and  $\rho$
  - Find S to get Loss < 10% using  $\rho$  and the M/ M/ 1/S result

Network Performance...

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## Network Performance Evaluation: Design Example

- Step 1: M/M/1 analysis to find  $R_{out}$ 
  - $\lambda = 55 * 2 = 110$  packets/sec
  - $E[D] = 100 \text{ ms} = 1/10 = 1/(\mu - \lambda)$
  - $\mu = 120$  packets/sec
  - $R_{out} = \mu * L = 120 * 9000 \text{ Bytes/packet} * 8 \text{ bits/Byte} = 8.64 \text{ Mb/s}$
- Step 2: M/M/1/S analysis to find system size(K)
  - $R_{in} = \text{Rate}_{in} = (110 \text{ packets/sec}) * 9000 \text{ Bytes/packet} * 8 \text{ bits/Byte} = 7.92 \text{ Mb/s}$
  - $\rho = R_{in} / R_{out} = (7.92 \text{ Mb/s}) / (8.64 \text{ Mb/s}) = \lambda / \mu = 110 / 120 = 0.916$
  - $\rho = 0.916$  and Blocking Prob = 0.1  $\rightarrow K = 7$
- Final design is:
  - $R = 8.64 \text{ Mb/s}$
  - Average system size  $\geq 7$  packets

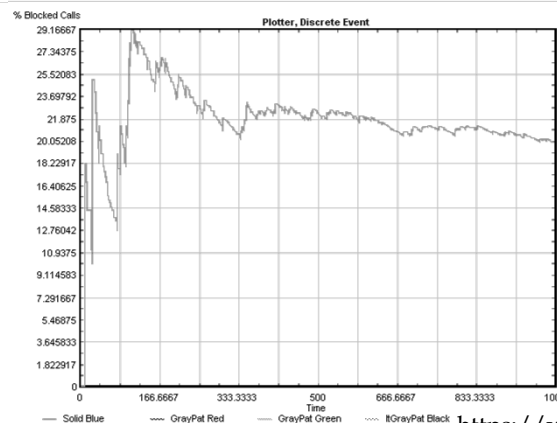
## Network Performance Evaluation: M/M/S/S

Erlang B

$$P[K = k] = \frac{\rho^k}{k!} \sum_{n=0}^S \frac{\rho^n}{n!}$$

$$P[K = k] = 0 \text{ for } k > S$$

$$P_{Blocking} = P[K = S] = \frac{\rho^S}{S!} \sum_{n=0}^S \frac{\rho^n}{n!}$$



Holding time=3min,  
Arrival rate=0.833 calls/min  
 $N = 4$

Load =  $3 * 0.833 = 2.5$  Erlangs

$\rightarrow$  Theory  $P_B = 0.15$   
(From Erlang B table)

Simulated  $P_B = 0.198$

<https://www.erlang.com/calculator/>

Show Extend Model:

[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/erlang-table.pdf](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/erlang-table.pdf)  
[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/Extend\\_Models\\_2019-v10/Telephone\\_Model-ES10.mox](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/Extend_Models_2019-v10/Telephone_Model-ES10.mox)

## Network Performance Evaluation: M/M/S/S

### Erlang B

$$P[K = k] = \frac{\rho^k}{k!} \frac{1}{\sum_{n=0}^S \frac{\rho^n}{n!}}$$
$$P[K = k] = 0 \text{ for } k > S$$
$$P_{\text{Blocking}} = P[K = S] = \frac{\rho^S}{S!} \frac{1}{\sum_{n=0}^S \frac{\rho^n}{n!}}$$

### Example

- S=9 (lines)
- Holding time = 3 min/call
- Arrival rate 2.16 calls/min
- Load = 3\*2.16=6.5
  - From Table Blocking Prob ~10%

<https://www.erlang.com/calculator/>

Show Extend Model:

[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/erlang-table.pdf](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/erlang-table.pdf)

[http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/Extend\\_Models\\_2019-v10/Telephone\\_Model-ES10.mox](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/Extend_Models_2019-v10/Telephone_Model-ES10.mox)

Network Performance...

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## Network Performance Evaluation: Example

- Design of a building phone system. The design goal is to minimize the number of lines needed between the building and the phone company. The blocking specification is  $P_{\text{blocking}} < 5\%$ .
- A building has four floors, on each floor is a separate department. Each department has 22 phones, each busy 10% of the time during the busy hour.

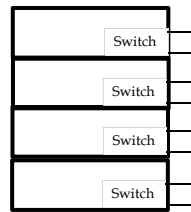
Network Performance...

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## Compare two designs

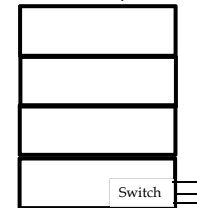
- Acquire one telephone switch for each floor and purchase lines from each separate switch to the phone company.
- Acquire one telephone switch for the building and purchase lines from the one switch to the phone company.

One Switch/Floor



Design Option A

One Switch/Building



Design Option B

Network Performance...

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## Network Performance Evaluation: Example-Case A

- Acquire one telephone switch for each floor.
- $22 \text{ phones} * 0.1 = 2.2 \text{ Erlangs/floor}$
- Use Erlang B table with 22 and  $P_{\text{blocking}} = 5\%$  to find  $S=5$
- 5 lines/floor or 20 lines for the building.

Network Performance...

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## Network Performance Evaluation: Example-Case B

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- Acquire one telephone switch for the building.
- 88 phones @ .1 Erlang/phone = 8.8 Erlangs
- 8.8 Erlangs & B=5% gives:  
    13 lines for the building
- Select Case B, Shared capacity
- Again → Aggregation/sharing improves system performance

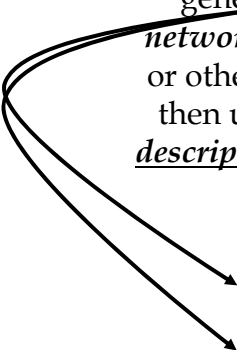
## Theoretical development of performance results

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- In extra slides and see....
- [http://www.ittc.ku.edu/~frost/EECS\\_563/LOCAL/EECS\\_563\\_Class\\_Notes-Fall-2021/Network\\_Performance\\_Analysis\\_2021.cdf](http://www.ittc.ku.edu/~frost/EECS_563/LOCAL/EECS_563_Class_Notes-Fall-2021/Network_Performance_Analysis_2021.cdf)

# Communication Network Simulation

Communication network simulation involves generating pseudo-random sequences representing network traffic (message lengths and interarrival times or other input processes, e.g. time varying link quality) then using these sequences to exercise an algorithmic description of the network operation to estimate system performance.



Message number	1	2	3	4	5	6	7	8	9	10	11	12
Interarrival time between i+1 and i message (seconds)	2	1	3	1	1	4	2	5	1	4	2	--
Length of i <sup>th</sup> message (seconds)	1	3	6	2	1	1	4	2	5	1	1	3

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# Verification and Validation of Simulation Models

- Model
  - Mathematical (Algorithmic) Description of Behaviour of “Real Thing”
- Verification
  - Determining Whether the Simulation Model Performs As Intended
  - In Programming Terminology, “Debugging”
  - Example: Is *Statistical Multiplexer* Model Producing the specified Message Lengths?
- Validation
  - Determining Whether the Simulation Model Itself Is an “Accurate” Representation of the Communication Network Under Study (the “Real Thing”)
  - Example: Is the Assumption the statistical message length model accurate?

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## Some Comments on Validation

---

- Simulation Models Are Always Approximations
- A Simulation Model Developed for One Application May Not Be Valid for Others
- Model Development and Validation Should Be Done Simultaneously
- Specific Modeling Assumptions Should Be Tested
- Sensitivity Analysis Should Be Performed
- Attempt to Establish That the Model Results Resemble the Expected Performance of the Actual System
- Generally, Validation Is More Difficult Than Verification

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## Quality of Performance Estimates

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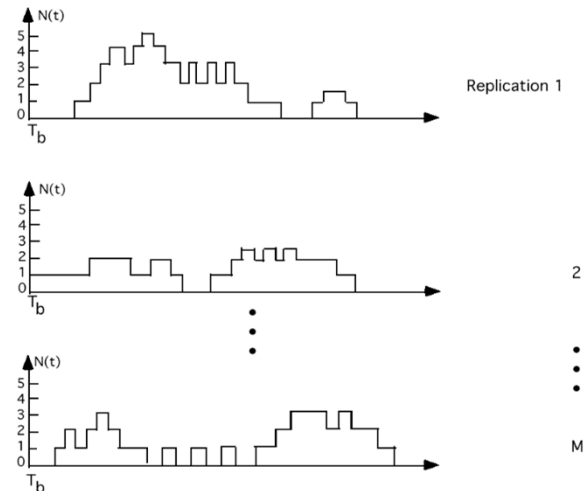
- Simulation results are like laboratory measurements, they can be modeled as random variables
- Performance estimates should have acceptable variance
- More observation reduces the variance.
- HOWEVER→ Often observations, e.g., a sequence of packet delays, taken from network simulation will be correlated
  - Cannot directly apply standard statistical approaches based on iid (Independent, Identically Distributed) observations

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# Dealing with Lack of Independence

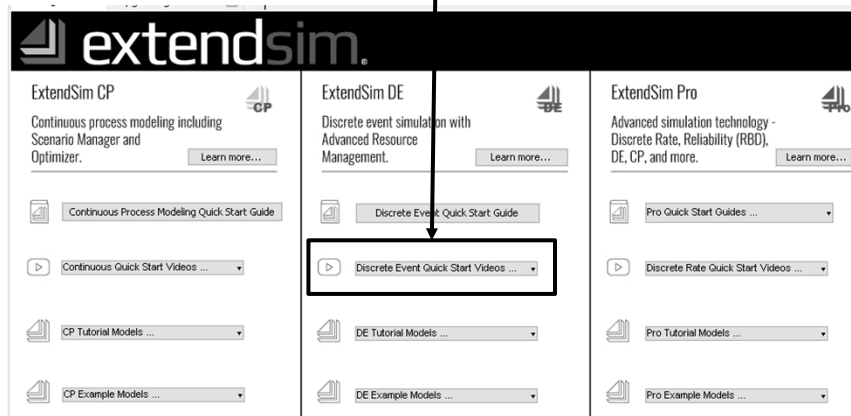
- Replication: Multiple Simulation Runs
  - Assume Results for Each Replication Are Independent
  - Can be Inefficient Because of Multiple Startup Periods
- Multiple simulation runs are commonly used to estimate the standard deviation of performance estimates and form confidence intervals



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# Extend®

- View Discrete Event Quick Start Videos



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## SoE has 25 Extendsim Licenses

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- Assume you hold a Extendsim license for 2 hours
- Requests for licenses come in at a rate of 11.5/hour
- Load = 23 Erlangs
- Probability you will be blocked from getting access to Extendsim =  $\sim 10\%$

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## Extra Slides

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# Network Simulation

## □ Outline

- Define network simulation
- Discuss attributes and application of simulation
- Present implementation of simulation systems
- Discuss analysis of simulation results
- Discuss selection of simulation tools
- Provide an overview of ExtendSim. On the start up Extendsim window there is:
  - A button for tutorials and a video showing how to build models
  - A link to "ExtendSim for DESS Textbook", a that is a tutorial on the tool.
  - Other useful tools.
  - There is a link to getting the whole user manual on the class web page. (It is long DO NOT PRINT the whole pdf file.)

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# A Definition of Communication Network Simulation

Communication network simulation involves generating *pseudo-random sequences representing network traffic* (message lengths and interarrival times or other input processes, e.g. time varying link quality) then using these sequences to *exercise an algorithmic description of the network operation.*



Message number	1	2	3	4	5	6	7	8	9	10	11	12
Interarrival time between i+1 and i message (seconds)	2	1	3	1	1	4	2	5	1	4	2	--
Length of i <sup>th</sup> message (seconds)	1	3	6	2	1	1	4	2	5	1	1	3

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## Attributes of Simulation

---

- Simulation Is a **Very Flexible** Evaluation Tool
  - General Network Characteristics (Sources, Topology, Protocols, Etc.)
  - Minute Detail
- Simulation Models Can Be **Expensive to Construct**
  - Human Effort
- Simulation Models Can Be **Expensive to Run**
  - Computer Effort
- Statistical **Analysis of the Results** Can Be Difficult
  - Requires Careful Interpretation
- **Difficult to Gain Insight** Into System Behavior
  - Simulate Only a Set of Specific Scenarios

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## When to Use Simulation

---

- Whenever Mathematical **Analysis Is Difficult or Impossible**
  - For Studying Transient Behavior of Networks
  - For Systems With Adaptive Routing
  - For Systems With Adaptive Flow Control
  - For Systems With Blocking (Finite Buffers)
  - For Systems With General Message Interarrival Statistics
- For **Validating Analytic Models** and Approximations
  - How Accurate Is the Model?
  - Do Approximations Distort the Results?
- For **Experimentation Without Disturbing** an Operational System
  - Test Possible Modifications and Adjustments

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## Modeling Elements for Communication Networks

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- Traffic and Input Processes
  - Message Arrival Process
    - Often Interarrival Times (probability density function)
  - Message Lengths (probability density function)
  - Other Message Attributes
    - Service Class
    - Error models
- Algorithmic Descriptions of Network Processing
  - Protocols
  - Links and Queues
  - Routing

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## Event Scheduling Approach to Network Simulation

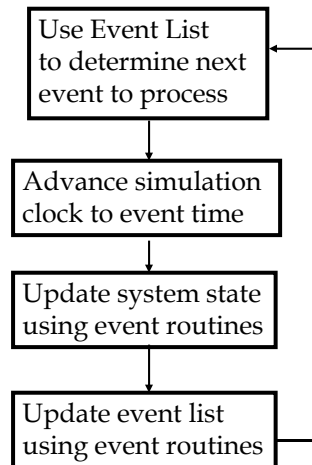
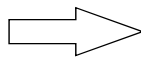
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- Variable Time Advance
  - Advance Time To Next Occurring Event
- Update System State Only When Events Occur
  - For Example, Arrivals or Departures
- Event Calendar
  - Events: Instantaneous Occurrences That Change the State of the System
  - An Event is Described by
    - The Time the Event is to Occur
    - The Activity to Take Place at the Event Time
  - The Calendar is a Time-Ordered List of Events

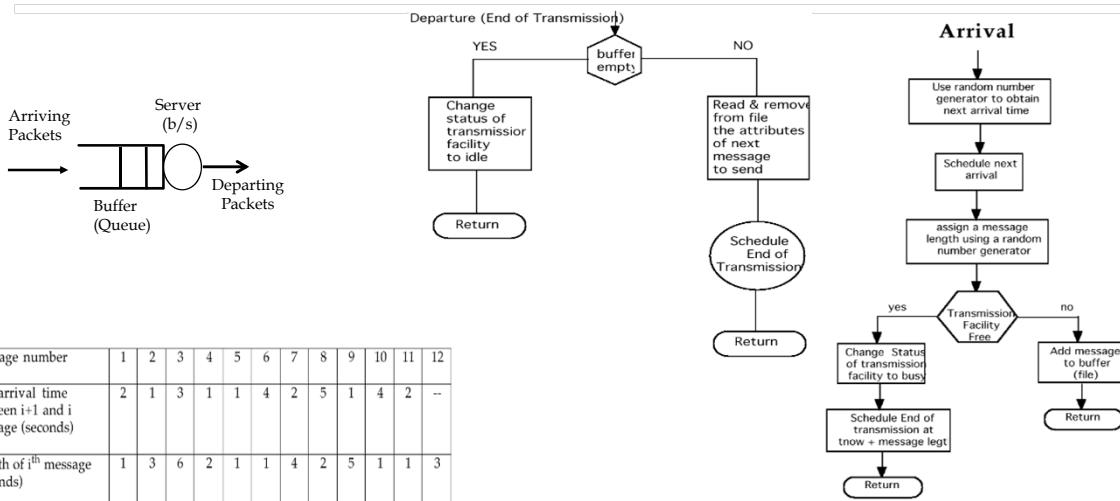
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# Event Scheduling Approach: Simplified Flow Control

An Executive  
(or Mainline)  
Controls the  
Selection of Next  
Event



# Event Scheduling for Simple Statistical Multiplexer



Message number	1	2	3	4	5	6	7	8	9	10	11	12
Interarrival time between $i+1$ and $i$ message (seconds)	2	1	3	1	1	4	2	5	1	4	2	--
Length of $i^{\text{th}}$ message (seconds)	1	3	6	2	1	1	4	2	5	1	1	3

# Verification and Validation of Simulation Models

---

- Model
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125

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- Specific Modeling Assumptions Should Be Tested
- Sensitivity Analysis Should Be Performed
- Attempt to Establish That the Model Results Resemble the Expected Performance of the Actual System
- Generally, Validation Is More Difficult Than Verification

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## Analysis of Results: Statistical Considerations

---

- Starting Rules
  - Overcoming Initial Transients
  - An Initial Transient Period Is Present Which Can Bias the Results
  - Achieving Steady State
    - Use a Run-in Period:
      - Determine  $T_b$  Such That the Long-Run Distribution Adequately Describes the System for  $t > T_b$
    - Use a “Typical” Starting Condition (State) to Initialize the Model
- Quality of Performance Estimates
  - Variance of Estimated Performance Measures

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## Quality of Performance Estimates

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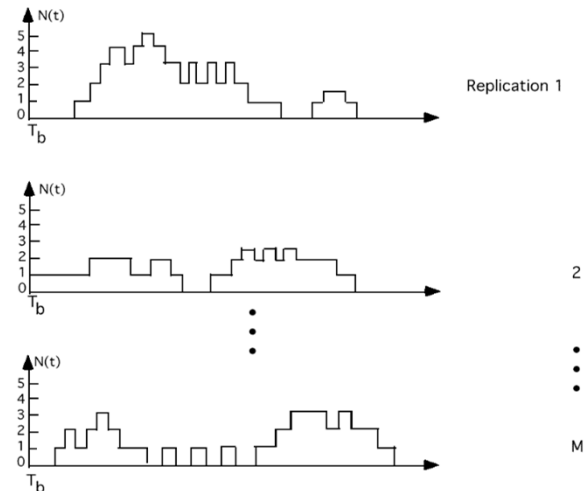
- Simulation results are like laboratory measurements, they can be modeled as random variables
- Performance estimates should have acceptable variance
- More observation reduces the variance.
- HOWEVER → Often observations, e.g., a sequence of packet delays, taken from network simulation will be correlated
  - Cannot directly apply standard statistical approaches based on iid (Independent, Identically Distributed) observations

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## Dealing with Lack of Independence

- Replication: Multiple Simulation Runs
  - Assume Results for Each Replication Are Independent
  - Can be Inefficient Because of Multiple Startup Periods



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## Criteria for Selecting a Network Simulation Tool

- Availability
  - Cost
  - Usage
  - Documentation
  - Ease of Learning
  - Computation Efficiency
  - Flexibility
  - Portability
  - User Interface
  - Extendibility
- Common Tools
    - ns-3  
(<http://www.nsnam.org/>)
    - Opnet  
(<http://www.opnet.com/>)
    - QualNet  
(<http://www.scalable-networks.com>)
    - ExtendSim

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## Guidelines to Network Modelling and Simulation

---

- Things to Know
  - Know the Customer
  - Know the Network
  - Know the Important Performance Metrics
- Things to Do
  - Establish a Credible Model
  - Expect the Model to Evolve → Plan for success
  - Apply Good Software Management Techniques

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## Conclusions

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- Simulation Can Be an Important Tool for Communication Network Design and Analysis
- Care and Thought Must Go Into Construction of Communication Network Models
- Care and Thought Must Go Into Interpretation of Model Output

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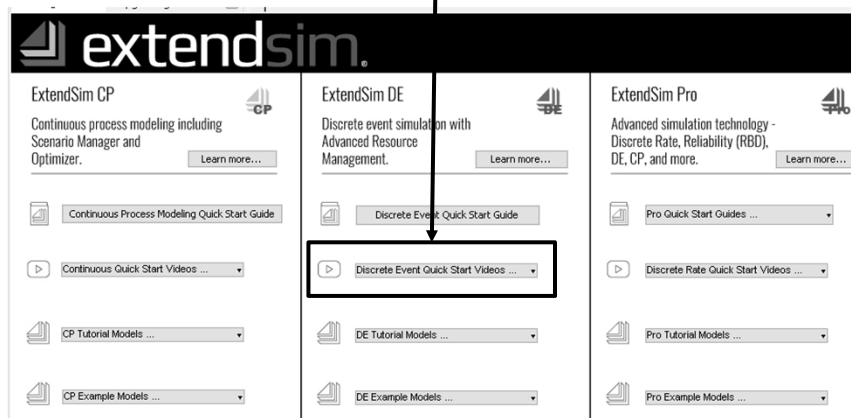
## Extend® Overview

- Allows Graphical Description of Networks
  - Sources, Links, Nodes, Etc.
- Data Flow Block Diagrams
- Hierarchical Structure to Control Complexity
- Be sure and create libraries when creating complex models

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## Extend® Overview

- View Discrete Event Quick Start Videos



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## Basic Queueing Theory

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## Network Performance Evaluation: Approach

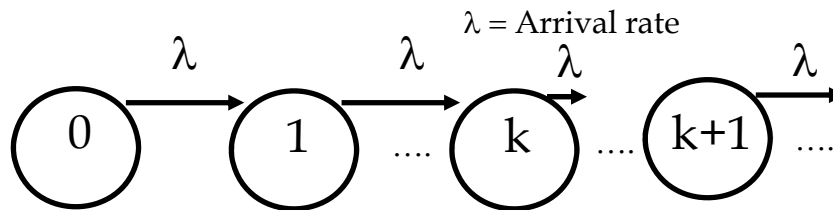
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- Analysis of a pure birth process to characterize arrival processes
- Extension to general birth/death processes to model arrivals and departures
- Specialization to the specific cases to find:
  - Probability of system occupancy,
  - Average buffer size,
  - Delay,
  - Blocking probability
- **Goal: Design and analyze statistical multiplexers and circuit switching systems**

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## Network Performance Evaluation: Analysis of a Pure Birth Process

Arrivals and no departures



Only Births (Arrivals) Allowed

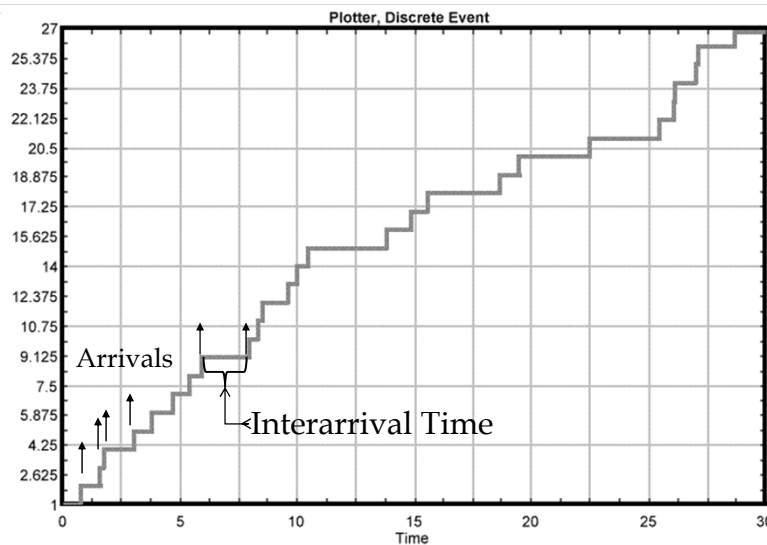
$k$  = System State (number in system)

- number of arrivals for 0 to  $t$  sec
- number in system at time  $t$

Goal: Find Prob [ $k$  arrivals in a  $t$  sec interval]

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## Network Performance Evaluation: Analysis of a Pure Birth Process



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## Network Performance Evaluation: Analysis of a Pure Birth Process

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- The number represents the *State* of the system. In networks this is usually the number in the buffer plus the number in service. *The system includes the server.*
- The time to clock the message bits onto the transmission facility is the service time. The server is the model for the transmission facility.
- Goal: Find Prob [k arrivals in a t sec interval]=P[k,t]

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## Network Performance Evaluation:

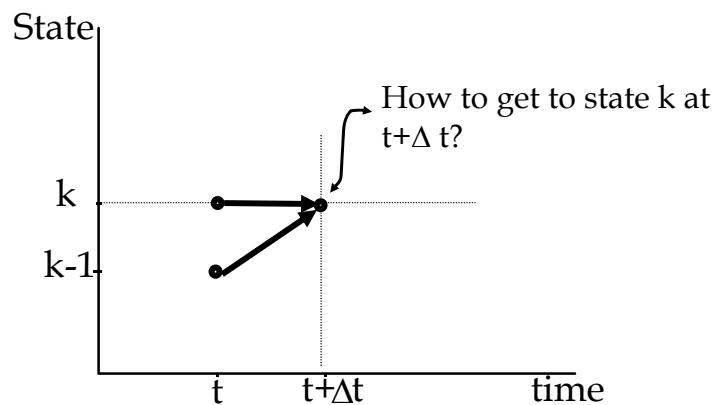
### Analysis of a Pure Birth (Poisson) Process: Assumptions

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- Prob[ 1 arrivals in  $\Delta t$  sec ] =  $\lambda \Delta t$
- Prob[ 0 arrivals in  $\Delta t$  sec ]  
=  $1 - \lambda \Delta t$
- Number of arrivals in non-overlapping intervals of times are statistically independent random variables, i.e.,  
Prob [ N arrivals in t, t+T AND M arrivals in t+T, t+T+ $\tau$  ] =  
Prob [ N arrivals in t, t+T ] \* Prob [ M arrivals in t+T, t+T+ $\tau$  ]

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## Network Performance Evaluation:



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## Network Performance Evaluation: Analysis

- Define probability of  $k$  in the system at time  $t = \text{Prob}[k, t]$
- Probability of  $k$  in the system at time  $t + \Delta t = \text{Prob}[k, t + \Delta t]$ 
  - $= \text{Prob}[k, t + \Delta t] \text{Prob}[(k \text{ in the system at time } t \text{ and } 0 \text{ arrivals in } \Delta t)$
  - or  $(k-1 \text{ in the system at time } t \text{ and } 1 \text{ arrival in } \Delta t)]$
  - $= (1 - \lambda \Delta t) \text{Prob}[k, t] + \lambda \Delta t \text{Prob}[k-1, t]$



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## Network Performance Evaluation: Analysis

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- Rearranging terms

$$(\text{Prob}[k, t + \Delta t] - \text{Prob}[k, t]) / \Delta t + \lambda \text{Prob}[k, t] = \lambda \text{Prob}[k-1, t]$$

- Letting  $\Delta t \rightarrow 0$  results in the following differential equation:

$$\frac{d\text{Prob}[k, t]}{dt} + \lambda \text{Prob}[k, t] = \lambda \text{Prob}[k-1, t]$$

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## Network Performance Evaluation: Analysis

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- For  $k = 0$  the solution is:

- $\text{Prob}[0, t] = e^{-\lambda t}$

- For  $k = 1$  the solution is:

- $\text{Prob}[1, t] = \lambda t e^{-\lambda t}$

- For  $k = 2$  the solution is:

- $\text{Prob}[2, t] = \frac{(\lambda t)^2 e^{-\lambda t}}{2}$

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## Network Performance Evaluation: Analysis

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- In general the solution is a Poisson probability mass function of the form:

$$\text{Prob } [k, t] = \frac{(\lambda t)^k e^{-\lambda t}}{k!}$$

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## Network Performance Evaluation: Analysis

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- A Poisson pmf of this form has the following moments:

$$E[k] = \lambda t$$

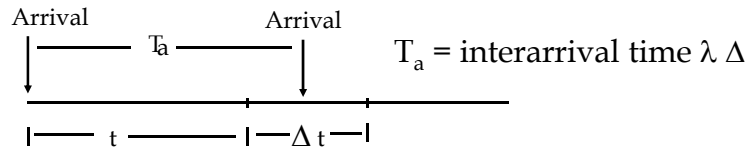
$$\text{Var}[k] = \lambda t$$

### **Poisson Arrival Process**

**The number of arrivals in any T second interval follows a Poisson probability mass function.**

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## Network Performance Evaluation: Interarrival Time Analysis



$\text{Prob}[t < T_a < t + \Delta t] = \text{Prob}[0 \text{ arrivals in } t \text{ sec and}$   
 $1 \text{ arrival in } \Delta t]$

$\text{Prob}[t < T_a < t + \Delta t] = \text{Prob}[k=0, t] \text{Prob}[k=1, \Delta t]$

$$\text{Prob}[t < T_a < t + \Delta t] = (e^{-\lambda t}) \lambda \Delta t e^{-\lambda \Delta t}$$

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## Network Performance Evaluation: Interarrival Time Analysis

Let  $\Delta t \rightarrow 0$  results in the following

$$\text{Prob}[t < T_a < t + dt] = f_{T_a}(t) dt = \lambda e^{-\lambda t} dt$$

so

$$f_{T_a}(t) = \lambda e^{-\lambda t} \text{ for } t > 0 \quad f_{T_a}(t) = 0 \text{ for } t < 0$$

$$P[T_a < t] = 1 - e^{-\lambda t}$$

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## Network Performance Evaluation: Interarrival Time Analysis

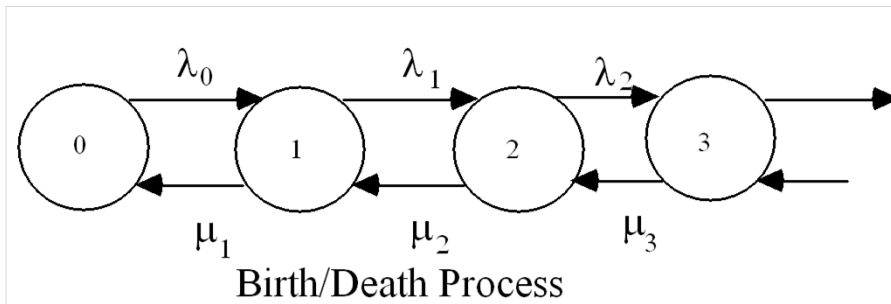
**MAIN RESULT:**  
The interarrival time  
for a Poisson arrival process follows  
an exponential probability density function.

$$E[T_a] = 1/\lambda \quad Var[T_a] = 1/\lambda^2$$

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## Network Performance Evaluation: Birth/Death Process Analysis

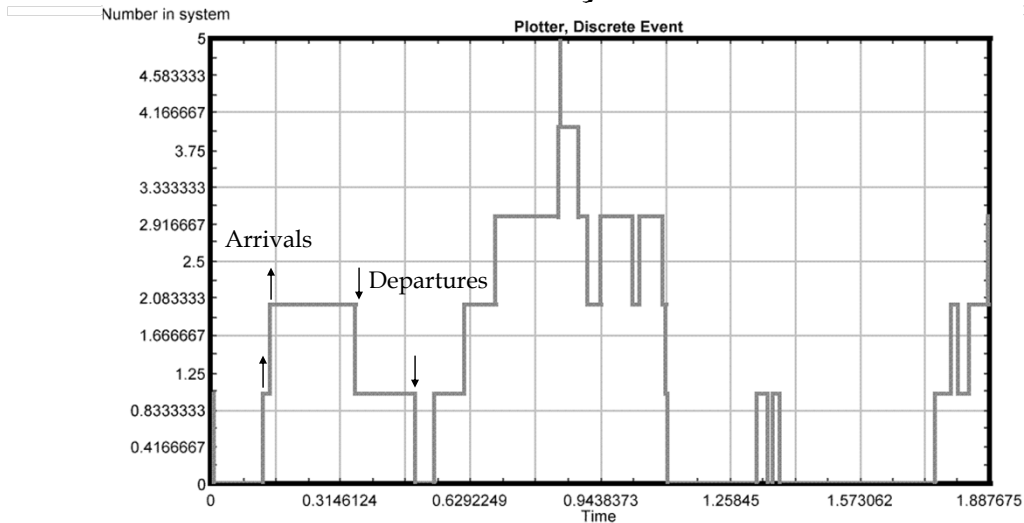
Now allow arrivals and departures.  
The Model for the Birth/Death Process



Note that the arrival and service rates are now state dependent

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## Network Performance Evaluation: Birth/Death Process Analysis



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## Network Performance Evaluation: Birth/Death Process Analysis

- The departure process is Poisson--
- Prob[ 1 departure in  $\Delta t$  sec when the system is in state  $k$  ] =  $\mu_k \Delta t$
- Prob[ 0 departure in  $\Delta t$  sec when the system is in state  $k$  ] =  $1 - \mu_k \Delta t$
- Number of departures in non-overlapping intervals of times are statistically independent random variables
- Probability[arrival AND departure in  $\Delta t$ ] = 0


152

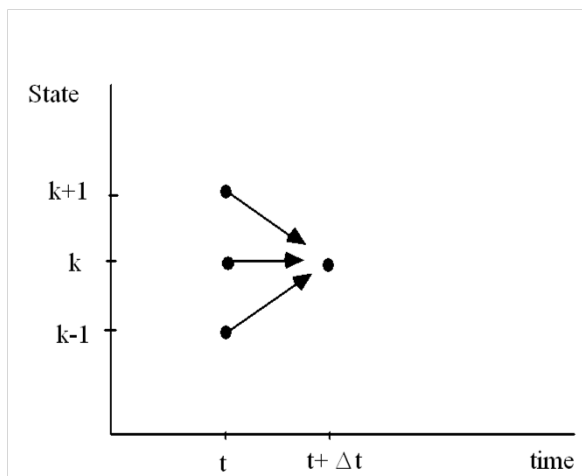
## Network Performance Evaluation: Birth/Death Process Analysis

**Poisson service process  
implies  
an exponential  
probability density function for the  
message length**

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## Network Performance Evaluation: Birth/Death Process Analysis

To solve for the state probabilities:  
Follow the procedure  
used for the pure birth  
process and use the  
transitions shown 



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## Network Performance Evaluation: Birth/Death Process Analysis

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- Specific queueing systems are modeled by
  - Setting state dependent arrival rates,  $\lambda_k$
  - Setting the state dependent service rates,  $\mu_k$
  - Solving for the steady state probabilities

For details see: Computer and Communication Networks, N. F. Mir: chapter 11 or Queueing Systems. Volume 1: Theory by Leonard Kleinrock, Wiley, 1975 (or any queueing theory book)

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## Network Performance Evaluation: Special cases: A / b / m / K / L

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- A = M => the arrival process is Poisson and the interarrival times are independent, identically distributed exponential random variables. (M = Markov)
- b = M => the service process is Poisson and the interdeparture times are independent, identically distributed exponential random variables.
- A or b = G => times are independent, identically distributed general random variables.
- A or b = D => times are deterministic, i.e., fixed times
- Examples:
  - M/M/1/∞/∞ (Ideal router output port)
  - M/M/1/S/∞ (Real-finite-buffer router output port)
  - M/M/S/S/∞ (Circuit Switch)

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## Network Performance Evaluation: M/M/1

- No limitation on buffer size means that the arrival rate is independent of state or  $\lambda_k = \lambda$
- Only one server means that the service rate is independent of state or  $\mu_k = \mu$

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## Network Performance Evaluation: M/M/1

Solving for  
the state  
occupancy  
probabilities

$$P[K=k] = \rho^k(1-\rho)$$

$$\text{With } \rho = \lambda L / C = R_{in} / C$$

The expected number in the systems is

$$E[K] = \frac{\rho}{1-\rho}$$

and the variance is

$$\text{Var} [K] = \frac{\rho}{(1-\rho)^2}$$

For M/M/1, if the load is greater than 1 then the systems is not stable and the buffer occupancy grows without bound.

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## Network Performance Evaluation: M/M/1/N

- Only one server means that the service rate is independent of state or  $\mu_k = \mu$
- The limitation on system size means that the arrival rate is dependent of state or

$$\lambda_k = \lambda \text{ for } k < N$$

$$\lambda_k = 0 \text{ for } k \geq N$$

Arrivals to a full system are blocked so there can be no arrivals to a full system.

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## Network Performance Evaluation: M/M/1/N

Solving for the state occupancy probabilities

$$P[k] = \frac{(1 - \rho)\rho^k}{1 - \rho^{N+1}} \text{ for } k \leq N$$

$$P[k] = 0 \text{ for } k > N$$

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## Network Performance Evaluation: M/M/1/N

- The Quality of Service (QoS) metric in this case is the probability of blocking.
- For a M/M/1/N queue the blocking probability is given by

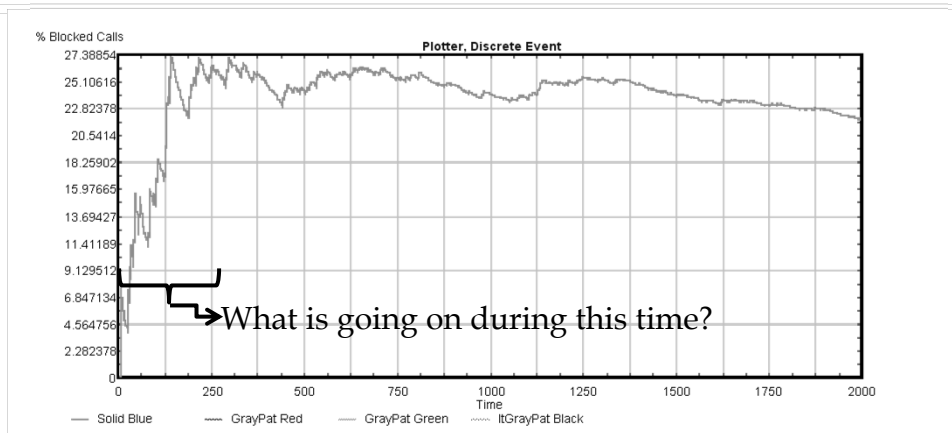
$$\text{Probability of Blocking} = P_B =$$

$$P_{k=N} = \frac{(1 - \rho)\rho^N}{1 - \rho^{N+1}}$$

Design Problem: Given  $P_B$  and  $\rho$  find  $N$   
(recommend constructing a spreadsheet)

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## Network Performance Evaluation: M/M/1/N



$N=3$

$\lambda=950$  &  $E[L]=1000$  bits,  $C=1\text{Mb/s}$  → Theory  $P_B=0.23$  Simulated  $P_B=0.219$

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## Network Performance Evaluation: M/M/S/N

The limitation on system size means that the arrival rate is dependent of state or

$$\lambda_k = \lambda \text{ for } k < N$$

$$\lambda_k = 0 \text{ for } k \geq N$$

Arrivals to a full system are blocked so there can be no arrivals to a full system.

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## Network Performance Evaluation: M/M/S/N

Multiple servers means that

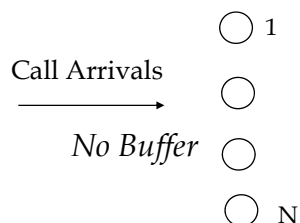
$$\mu_k = k\mu \text{ for } k \leq S$$

$$\mu_k = S\mu \text{ for } k > S$$

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## Network Performance Evaluation: M/M/S/N

This model is difficult to solve in general.  
 The case of special interest is  $S=N$ : the M/M/N/N case.  
 This case models a circuit switch system with  $N$  transmission facilities. A call arriving to the system with all transmission facilities busy is blocked.



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## Network Performance Evaluation: Erlang B formula

Solving for the state occupancy probabilities

$$P[k] = \frac{\rho^k}{\sum_{n=0}^N \frac{\rho^n}{n!}} \quad k=0 \dots N$$

and

Relationship among  $P_B$ ,  $N$ ,  $\rho$   
 found using provided table  
 or web Erlang B calculator

$$P_B = P[k = N] = \frac{\frac{\rho^N}{N!}}{\sum_{n=0}^N \frac{\rho^n}{n!}}$$

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